ORIGINAL WORKS

PARAMETRIC EXCITATION OF ELECTRIC OSCILLATIONS
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The article describes an approximate theory of the phenomena of oscillation excitation in electric oscillation system, where there are no obvious sources of electric or magnetic forces. The theory is based on periodic change in electric oscillation system parameters. It is rooted to the general methods (previously developed by Poincare) of finding periodic solutions to differential equations. The special cases of such excitation with the sinusoidal self-induction and capacity change in an oscillatory system with one degree of freedom, as well as with self-induction change in a regenerated system are considered here in a detail. The article describes the experiments for generating oscillations with mechanical parameter change in the system with regeneration as well as without it. These experiments prove a possibility of such excitation and are in agreement with the theory.

The phenomenon of oscillation excitation by means of periodic change in the oscillatory system parameters well-known in physics already for a long time [Melde ([1]), Rayleigh ([2],[3],[4]) and others ([5])] becomes currently interesting due to realization of such excitation in electric oscillatory systems. Although there were some indications of such excitation possibility (which we will briefly call "parametric excitation") ([3], [6]) and it undoubtedly plays a significant but not always a clearly realized part, as, for instance, in case of normal current generation in the electric engineering, however, it was performed deliberately and systematic study has begun. Hegner ([8]) and later Gunther-Winter ([9]) described experiments on oscillation excitation in the electric oscillatory system in the field of acoustic frequencies by means of periodic magnetization of a self-inductor iron core. Afterwards, using the change of self-induction formed by the series connection of two phases both of the stator and rotor of three-phase generator during the rotor rotation Gunther-Winter ([10]) also performed the parametric oscillation excitation. Quite lately there appeared descriptions of I. Watanabe, T. Saito and I. Kanto`s ([11]) experiments on oscillation excitation by means of mechanical periodic change in the magnetic circuit of the system self-induction.

We started the theoretical and experimental research on parametric oscillation excitation issues in 1927 (at NIIF (Research Institute of Physics) in Moscow and at the CRL (Central Radio Laboratory) and first we received and examined the oscillation excitation phenomenon (up to frequencies about 10^6 Hz) with a periodic change of an iron core magnetization of the system self-induction ([12]). Later in LEFI (Electrophysical Measurements Laboratory) we studied the parametric excitation phenomena with mechanical change in parameters ([12], [13]), but we delayed publication of the results until now due to the patent reasons. As it is pointed in our article in TPJ, Volume III, №

7, 1933, (References are given at the end of the work) besides the parametric oscillation excitation by means of mechanical self-induction change performed in early 1931, in LEFI we have recently received parametric excitation by means of mechanical change in capacity as well ([16]). As for the theory of parametric excitation phenomena it should be noted that we already have necessary preconditions for a complete analysis of oscillation excitation conditions provided in other scientific works. This issue as we know leads to the research of the so-called "unstable" solutions of linear differential equations with periodic coefficients, which are mathematically quite thoroughly researched in general and specifically in terms of the problem being discussed [Rayleigh ([2], [8]), Andronov and Leontovich ([14]), van der Pol and Strutt ([15])]. However, the theory of these equations based on the linear ones cannot answer the questions about the value of the stationary amplitude, its stability, the process of setting, etc., adequate interpretations of these issues are possible only by means of nonlinear differential equations. The authors mentioned above (Gunther-Winter, Watanabe) stick only to a simplified conclusion on oscillation conditions based on the analysis of a corresponding linear differential equation and leave the question about the stationary amplitude unanswered. However, these problems are no less fundamental than the question about the oscillation excitation and the solution of which is necessary not only for a complete description of all phenomena, but also to make any calculations in this field possible.

This article describes the approximate theory of the process of parametric oscillation excitation based on common methods of finding periodic solutions of differential equations given by Poincare. This work deals with the cases of periodically changeable self-induction and capacity, as well as some results of experiments made in 1931 and 1932 in LEFI. Other related experimental and theoretical information is represented in the articles by V. A. Lazarev, V.P. Gulyaev and V. V. Migulin provided below.

The results of more detailed experimental research of the parametric excitation phenomena by means of periodic change in magnetization of self-induction core performed in CRL are provided in other works.

In this paper we confine ourselves to considering in practice only the first approximation, perhaps, of the most significant case of parametric excitation, when frequency of the parameter change is approximately two times greater than the average proper frequency of the system. However, the methods used in this work allow making solution of the problem for other cases as well as finding further approximations. Some similar issues would be considered separately.

THEORETICAL PART

§ 1. On oscillation onset with parametric excitation.

Some general arguments and conclusions

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As we showed in the previous studies ([13], [16]), based on energy considerations it is easy to understand the physical side of the oscillation excitation process by means of periodic (abrupt) changes in capacity of the system, which do not contain any obvious sources of magnetic or electric fields.

Let us briefly remind this argument for the case of self-induction change. Suppose there is current i in the oscillatory system having capacity C, ohmic resistance R and induction L at a period of time taken as the initial one. Let us change self-induction to the magnitude ΔL at this moment, which is equivalent to energy increase equal to $\frac{1}{2}\Delta Li^2$. Now we leave the system to itself. In a period of time equal to ¼ of the system proper oscillations period, the entire system energy will transform from magnetic into electrostatic. At this moment, when the current = zero, we return the self-induction to its initial magnitude, which obviously can be performed without an effort, and then leave the system to itself again. In the next ¼ of the proper oscillation period the electrostatic energy will entirely transform into the magnetic one again, and then we can start a new cycle of induction change. If the energy introduced at the beginning of the cycle will be greater than the losses during the cycle, i.e., if

$$\frac{1}{2}\Delta Li^2 > \frac{1}{3}Ri^2\frac{T}{2}$$

or

$$\frac{\Delta L}{L} > \varepsilon$$

where ε is a logarithmic decrement of the proper system oscillations, then the current at the end of each cycle will be greater than at the beginning. Thus, repeating these cycles, i.e. changing self-induction with frequency that is twice as large as the average proper frequency of the system so that

$$\frac{\Delta L}{L} > \varepsilon,$$

it is possible to excite oscillations in the system with no affecting of any electromotive force, no matter how small a random initial charge is. Note that even without any random induction that almost always inevitably occur (electric line, Earth`s magnetic field, atmospheric charges), we fundamentally should always have random charges in the loop because of statistical fluctuations.

Even having such a gross, rather qualitative analysis of the phenomena of oscillation excitation it is possible to derive two basic preconditions for its occurrence: 1) the need to achieve a specific relation between the frequency of the parameter changes and the "average" natural frequency of the system and 2) the need to keep to a certain relation between the magnitude of the relative parameter change - the so-called *modulation depth* and the magnitude of the average logarithmic decrement of the system.

The more profound analysis of oscillation phenomena at parametric excitation leads to the linear differential equations with periodic coefficients. For example, in case of change of the system capacity according to the law:

$$\frac{1}{C} = \frac{1}{C_0}(1 + m\sin \gamma t) \tag{1}$$

we have the following equation for $q = \int i\,dt$:

$$L\frac{d^2 q}{dt^2} + R\frac{dq}{dt} + \frac{1}{C_0}(1 + m\sin \gamma t)q = 0 \tag{2}$$

which by means of the transformation

$$q = xe^{-\frac{R}{2L}t} \tag{3}$$

can be reduced to:

$$\ddot{x} + \lambda^2(1 + m_1 \sin 2\tau)x = 0, \tag{4}$$

where

$$\ddot{x} = \frac{d^2 x}{d\tau^2}, \quad \tau = \frac{\nu t}{2}, \quad \omega_0^2 = \frac{1}{LC_0}, \quad 2\delta = \frac{R}{L}, \quad \omega_1^2 = \omega_0^2 - \delta^2,$$

$$m_1 = \frac{m\,\omega_0^2}{\omega_1^2}, \quad \vartheta = \frac{2\delta}{\nu}, \quad \lambda^2 = \frac{4\,\omega_1^2}{\nu^2}. \tag{5}$$

Hence in the concerned case the mathematical problem is reduced to a simple linear second-kind differential equation with periodic coefficients (4), known as *Mathieu equation* ([14], [15]). Note that many other problems are reduced to these types of equations: in astronomy, optics, elasticity theory, acoustics, etc. From the mathematic side they are well studied by *Mathieu*, *Hill*, *Poincare*, etc.

As it is known the solution of the equation (4) can be represented as:

$$x = C_1 e^{hx}\chi(\tau) + C_2 e^{-hx}\chi(-\tau) \tag{6}$$

where $\chi(\tau)$ is a periodic function with the period π (or 2π).

Inserting this solution into (3) we obtain for *q*:

$$q = C_1 e^{(h-\vartheta)\tau}\chi(\tau) + C_2 e^{-(h+\vartheta)\tau}\chi(-\tau) \tag{7}$$

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It follows from this expression that the problem of oscillation excitation is ended in finding the conditions, under which the amplitude q will increase consistently. We can see from the equation (17) that this will take place when the real part h is absolutely more than 0. Therefore the condition of parametric excitation is closely linked to the magnitude h, i.e. to the characteristic exponent of the Mathieu equation solution (4). Dependence of h on the parameters of this equation m and $\lambda = \dfrac{2\omega_1}{v}$ can be qualitatively figured (Fig. 1), as did Andronov and Leontovich ([14]), having distinguished the areas, within which h has a real part, separately at the plane $\left(m, \dfrac{2\omega_1}{v}\right)$.

As the figure shows, these areas that are the areas of "unstable" solutions of the equation (4) are located near the values $\dfrac{2\omega_1}{v} = 1, 2, 3\ldots$ Having the damping, i.e. for the equation (2) these areas of instability are greatly reduced (dashed areas in Fig. 1).
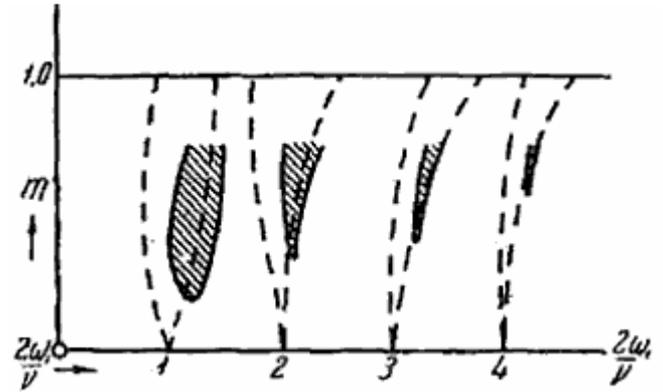
Fig. 1. Instability areas (by Andronov and Leontovich).

Using the method described by Rayleigh ([3], [4]), it is possible to determine approximately the boundaries of these instability areas. Thus the boundaries of the first instability area (about the value $\dfrac{2\omega_1}{v} = 1$) are given as curves up to m^2:

$$\frac{2\omega_1}{v} = \sqrt{1 + \sqrt{\frac{m^2}{4} - 4\vartheta^2}} \quad \text{and} \quad \frac{2\omega_1}{v} = \sqrt{1 - \sqrt{\frac{m^2}{4} - 4\vartheta^2}} \tag{8}$$

This means that having the defined m and ϑ and the values $\dfrac{2\omega_1}{v}$ that satisfy the inequations

$$\sqrt{1 + \sqrt{\frac{m^2}{4} - 4\vartheta^2}} \geq \frac{2\omega_1}{v} \geq \sqrt{1 - \sqrt{\frac{m^2}{4} - 4\vartheta^2}} \tag{9}$$

the solution of the equation (2) is "unstable".

It is necessary to take account of the members m^4 to determine the second "instability" area (about $\dfrac{2\omega_1}{v} = 2$). As shown by Andronov and Leontovich ([14]) in this case we have:

$$\sqrt{4 + \frac{2}{3}m^2 + \sqrt{m^4 - 64\vartheta^2}} \geq \frac{2\omega_1}{v} \geq \sqrt{4 + \frac{2}{3}m^2 - \sqrt{m^4 - 64\vartheta^2}} \tag{10}$$

Hence the magnitude (width) of the "instability" area is depressed with its n as m^n.

The conditions (9) and (10) contain consequently the following additional conditions.

For the first instability area:

$$\frac{m^2}{4} > 4\vartheta^2 \quad \text{or} \quad m > 4\vartheta,\tag{11}$$

for the second one

$$m^4 > 64\vartheta^2 \quad \text{or} \quad m > 2\sqrt{2\vartheta}\tag{12}$$

The equations (11) and (12) show that the condition of parametric excitation, with approximate setting of the system to a frequency that is equal to the frequency of parameter change, is much harder to fulfill than the excitation condition with setting of the system to a half-frequency, since it requires much greater depth of parameter modulation parameter m under given damping. There are even more severe conditions for parametric excitation under the frequency relation like $\frac{2\omega_1}{v} = 2, 3...$ etc. Therefore the case of $\frac{2\omega_1}{v} = 1$ is of more interest, which is almost entirely considered in this work.

As it is shown above, the question of oscillation excitation conditions under a parametric stimulus is solved by means of the formulas (9) and (11). On the one hand, those specify the conditions that damping of the system must satisfy, in order that waves could occur in it under the given parameter change, but on the other hand, they show the extent of changes that we can make in system resistance (load) or the system detuning due to the exact parametric resonance, without compromising the possibility of oscillation excitation. However, these formulas do not and cannot answer the question of whether the stationary oscillation amplitude is settled and what value it has. In fact, the original equation (2) as a linear equation cannot answer this question. In other words, if the system is genuinely governed by this equation all the time, the oscillation amplitude will increase with no limit under the conditions (9).

Hence a linear system cannot be an alternator. In order to set a stationary amplitude in the system, it is necessary to make it be governed by a nonlinear differential equation. The equation (2) that was considered may be only approximate for a finite amplitude interval. It remains the full meaning here and allows us to solve the question of oscillation excitation.

The experiences described below also confirm that the phenomenon occurs the defined way. Without adding nonlinearity to the oscillation system, under periodic changes in its parameters we can see the following. As soon as the excitation conditions are observed current occurs in the loop whose amplitude increases constantly. In our experiences this increase reached the stage when the insulation of the capacitor or lead wires could not stand and we had to stop.

We had to add a nonlinear conductor to the system to obtain a stationary condition, like an iron-core coil, incandescent lamps, etc. Mathematically, in case of adding iron-core coil to the considered system we deal with the equation:

$$\frac{d\Phi(i)}{dt} + Ri + \frac{1 + m\sin\nu t}{C_0}\int idt = 0$$

where the nonlinear relation between current and magnetic flux in the loop Φ (*i*) is a certain specified function *i*, e.g. in the form of a power series.

Since the question is the theory of the observed phenomena, we need to investigate precisely this kind of nonlinear equations, moreover mathematically we have a two-fold task here: on the one hand it is required to find conditions, under which the equilibrium position of the system becomes unstable (oscillation excitation condition) and on the other hand it requires to find and investigate properties of periodic solutions of this equation (value of stationary amplitude, conditions of its stability, etc.). In the next section we consider this problem in a number of examples.

§ 2. Formulation of the problem for particular cases

Let us formulate the problem of oscillation excitation mathematically by means of a periodic change of the oscillation system parameter for a number of particular cases. First we will consider the following simple case. Let us have a circuit with total ohmic resistence R consisting of capacity C and two self-induction coils as an oscillation system. Let us suppose that one of the coils is a specified harmonic function of time:

$$L_1 = L_{10} + l_1\sin 2\omega t,$$

and the other coil is a some kind of reactor choke with a core of partitioned iron with very low hysteresis losses, so that the relationship between the magnetic flux through the coil and the current in it will be given as a unique function $\varphi(i)$, such as an *n*-degree polynomial of *i*.

For instance, the simplest case may be:

$$\varphi(i) = C + \alpha i + \beta i^2 + \gamma i^3 \qquad (13)$$

Then the instantaneous value of the magnetic flux in the circuit is expressed in terms of:

$$\Phi = L_1 i + \varphi(i) \qquad (14)$$

Anf hence the differential equation of the problem can be written as:

$$\frac{d}{dt}[L_1 i + \varphi(i)] + Ri + \frac{1}{U}\int idt = 0 \qquad (15)$$

whence, we assume,

$$\int idt = q$$

and after differentiation we obtain:

$$\left(\varphi'(\dot q) + L_{10} + l_1 \sin 2\omega t\right)\ddot q + \left(R + 2\omega l_1 \cos 2\omega t\right)\dot q + \frac{1}{C}q = 0$$

or taking into account (13) we obtain:

$$\left(L_{10} + \alpha + l_1 \sin 2\omega t + 2\beta \dot q + 3\gamma \dot q^2\right)\ddot q +$$
$$+ \left(R + 2\omega l_1 \cos 2\omega t\right)\dot q + \frac{1}{C}q = 0. \qquad (16)$$

Hence the problem of parametric excitation leads to a nonlinear second order differential equation with periodic coefficients, which can not be solved in a general form. However, in cases when: 1) l_1 and the variable (depending on q) component $\varphi`(q)$ are small in comparison with $L_{10} + \alpha$ and 2) the eigen "average" logarithmic decrement of the circuit is small in comparison with one, it is possible to bring this equation to:

$$\ddot x + x = \mu f(x, \dot x, \tau, \mu), \qquad (17)$$

where μ is a a "small" parameter of the equation, and apply Poincare methods to finding its periodic solutions.

Truly let us transform the equation 16.

Introducing the new time scale

$$\tau = \omega t$$

and assuming that:

$$\left.\begin{array}{c} L_0 = L_{10} + \alpha, \quad \dfrac{l_1}{L_0} = m, \quad 2\vartheta = \dfrac{R}{L_0 \omega}, \quad \omega_0^2 = \dfrac{1}{L_0 C} \\[2mm] \xi = \dfrac{\omega^2 - \omega_0^2}{\omega^2}, \quad \beta_1 = \dfrac{2\beta \omega q_0}{L_0}, \quad \gamma_1 = \dfrac{3\gamma \omega^2 q_0^2}{L_0}, \quad x = \dfrac{q}{q_0}, \\[2mm] \dot x = \dfrac{dx}{d\tau}, \quad \ddot x = \dfrac{d^2 x}{d\tau^2} \end{array}\right\} \qquad (18)$$

we obtain:

$$(1 + m \sin 2\tau + \beta_1 \dot x + \gamma_1 \dot x^2)\ddot x + 2(\vartheta + m \cos 2\tau)\dot x + (1 - \xi) x = 0. \qquad (19_1)$$

instead of (16).

According to the assumptions m, β_1, γ_1, ϑ and ε are small in comparison with with one. This condition can be expressed somewhat differently, having denoted the greatest of these values (in absolute magnitude) through μ in such a way that:

$$\frac{m}{\mu}, \frac{\beta_1}{\mu}, \frac{\gamma_1}{\mu}, \frac{\vartheta}{\mu} \text{ and } \frac{\xi}{\mu}$$

must be less than one, where

$$\mu \ll 1.$$

Hence we can assume:

$$\left.\begin{array}{c} m \sin 2\tau + \beta_1 \dot x + \gamma_1 \dot x^2 = \mu \chi(\dot x, \tau) \\[2mm] (m \sin 2\tau + \beta_1 \dot x + \gamma_1 \dot x^2 + \xi) x - 2(\vartheta + m \cos 2\tau)\dot x = \mu \psi(x, \dot x, \tau), \end{array}\right\} \qquad (20)$$

so that the equation (19_1) can be written as:

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$$\ddot{x} + x = \frac{\mu \psi(x, \dot{x}, \tau)}{1 + \mu \chi(\dot{x}; \tau)} = \mu f(x, \dot{x}, \tau, \mu). \tag{21}$$

Here, as we can see from (20), $f(x`, x, \tau, \mu)$ is a periodic function of τ with the period π.

Thus we draw the conclusion that in the considered case the question of oscillation excitation by means of periodic change in self-induction of the oscillation system is reduced to solving an equation of the type (21), to which the methods used in our work "N-th type resonance" ([17], [18]) may be applied.

Before turning to the approximate solution of this equation let us consider some other cases of parametric excitation, which we have been dealing with during the experiments and the theory of which leads to the same differential equation.

Under sinusoidal change in capacity, e.g. according to the law:

$$\frac{1}{C} = \frac{1 + m \sin 2\omega t}{C_0}$$

and having the reactor choke with the considered above relationship between the magnetic flux and the current in the system, we have the following differential equation:

$$(L_{10} + \alpha + 2\beta\dot{q} + 3\gamma\dot{q}^2)\ddot{q} + R\dot{q} + \frac{1 + m \sin 2\omega t}{C_0}q = 0 \tag{16_1}$$

or introducing the notation (18) we obtain:

$$(1 + \beta_1\dot{x} + \gamma_1\dot{x}^2)\ddot{x} + 2\vartheta\dot{x} + (1 + m \sin 2\tau)(1 - \xi)x = 0, \tag{19_2}$$

where we have again:

$$\ddot{x} + x = \frac{\mu \psi(x, \dot{x}, \tau)}{1 + \mu \chi(\dot{x})},$$

where

$$\left.\begin{aligned}
\mu \chi(\dot{x}) &= \beta_1\dot{x} + \gamma_1\dot{x}^2, \\
\mu \psi(x, \dot{x}, \tau) &= [\xi - m(1 - \xi)\sin 2\tau + \beta_1\dot{x} + \gamma_1\dot{x}^2]x - 2\vartheta\dot{x}.
\end{aligned}\right\} \tag{20_1}$$

Now let us consider the case of self-induction change in the regenerated system. As a typical regenerated system let us take a usual tube cyclic circuit with oscillation contour in the grid circuit (Fig. 2). Here we have the following differential equation for the oscillation circuit:

$$\frac{d}{dt}[L_0(1 + m \sin 2\omega t)\dot{q}] + R\dot{q} + \frac{1}{C}q = M\frac{di_a}{dt}. \tag{22}$$

Here

$$L_0 = L_{10} + L_2,$$

where L_2 is a coefficient of the closed loop coil self-induction, and L_{10} is a constant part of the periodically changiing self-induction, like in the case considered above.

Hence here
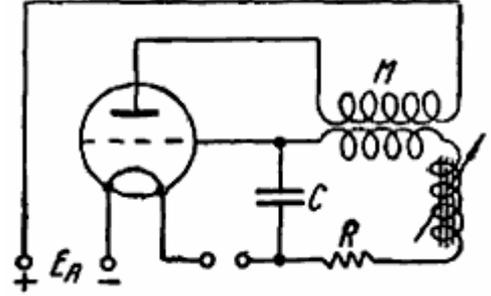
$$m = \frac{l_1}{L_2 + L_{10}}$$

Fig. 2. The scheme of the regenerative system.

Considering that the lamp has a very low transmittivity it is possible to assume i_a as a function of only one grid voltage and then, for instance, as an n-th degree polynomial of q. We confine ourselves to the simplest case, where:

$$i_a = i_{a0} + \alpha q + \beta q^2 + \gamma q^3 \qquad (23)$$

Assuming that

$$\rho = \frac{M}{L_0}, \ \alpha_1 = \alpha\rho, \ 2\beta\rho q_0 = \beta_1, \ 3\gamma\rho q_0^2 = \gamma_1 \ \text{and} \ \alpha_1 - 2\vartheta = k \qquad (18_1)$$

we have:

$$(1 + m\sin 2\tau)\ddot{x} + 2(\vartheta + m\cos 2\tau)\dot{x} + (1 - \xi)x = (\alpha_1 + \beta_1 x + \gamma_1 x^2)\dot{x},$$

whence we have the equation (21) again, where

$$\psi(x, \dot{x}, \tau) = (k + \beta_1 x + \gamma_1 x^2 - 2m\cos 2\tau)\dot{x} + (\xi + m\sin 2\tau)x$$

$$\chi(x, \dot{x}, \tau) = \dot{m}\sin 2\tau. \qquad (20_2)$$

As the last example consider a system that consists of an oscillation loop inductively linked to a nonperiodic circuit, besides let the mutual induction between circuit and loop be the parameter that changes periodically. This scheme basically corresponds the setting for a periodic change of self-induction described in the experimental section.

In this case differential equations of the problem can be written as:

$$\frac{d\Phi}{dt} + R_1 i_1 + \frac{1}{C}\int i_1 dt = -\frac{d}{dt}(Mi_2)$$

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$$\frac{d(L_2 i_2)}{dt} + R_2 i_2 = -\frac{d}{dt}(M i_1)$$

If $R_2 = 0$, this equation system can be replaced by a single equation:

$$\frac{d\Phi}{dt} + R_1 i_1 + \frac{q_1}{C} = -\frac{d}{dt}\left(\frac{M^2 i_1}{L_2}\right) \qquad (15_1)$$

Let us analyze this equation closer in two particular cases.

A)
$$L_2 = const \quad M = M_0(1 + m\sin 2\omega t)$$
$$\Phi = L_1 i_1 + \beta i_1^2 + \gamma i_1^3$$

In this case we have

$$\left(1 + m_1\left(\sin 2\tau - \frac{m}{4}\cos 4\tau\right) + \beta_1 \dot{x} + \gamma_1 \dot{x}^2\right)\ddot{x} +$$
$$+ 2\left[\vartheta + m_1\left(\cos 2\tau + \frac{m}{4}\sin 4\tau\right)\right]\dot{x} + (1 - \xi) x = 0,$$

where
$$L_0 = L_1 - \frac{M_0^2}{L_2}\left(1 + \frac{m^2}{2}\right), \quad m_1 = \frac{2M_0^2 m}{L_2 L_0}, \quad \omega_0^2 = \frac{1}{L_0 C}.$$

Hence here

$$\mu \cdot \psi(x, \dot{x}, \tau) = \left[m_1\left(\sin 2\tau - \frac{m}{4}\cos 4\tau\right) + \beta_1 \dot{x} + \gamma_1 \dot{x}^2\right] x +$$
$$+ \xi x - 2\left[\vartheta + m_1\left(\cos 2\tau + \frac{m}{4}\sin 4\tau\right)\right]\dot{x},$$

$$\mu\chi(\dot{x}, \tau) = m_1\left(\sin 2\tau - \frac{m}{4}\cos 4\tau\right) + \beta_1 \dot{x} + \gamma_1 \dot{x}^2.$$

Comparing these formulas with the (20) we see that they differ only in presence of members containing cos4τ and sin4τ , which, as seen below, do not play any part in the first approximation when finding the "zero" solution.

B)
$$L_2 = L_{20}(1 + m\sin 2\omega t), M = M_0(1 + m\sin 2\omega t), \Phi = L_1 i_1 + \beta_1 i_1^2 + \gamma_1 i_1^3.$$

Since in this case

$$\frac{M^2}{L_2} = \frac{M_0^2}{L_{20}}(1 + m\sin 2\omega t)$$

the equation (15_1) is brought to quite the same form as the equation (15).

§ 3. Finding periodic solutions of the equation (21)

As it was already pointed finding periodic solutions of the equation (21) can be performed by means of the methods developed in the works stated above ([17], [18]).

Using this method it is possible by means of the substitution:

$$x = u \sin \tau - v \cos \tau \\ \dot{x} = u \cos \tau + v \sin \tau \qquad (24)$$

to replace this equation by a system of two first-order equations:

$$\dot{u} = \mu f(u, v, \tau, \mu) \cos \tau \\ \dot{v} = \mu f(u, v, \tau, \mu) \sin \tau \qquad (25)$$

Here

$$f(u, v, \tau, \mu) = \frac{\psi(x, \dot{x}, \tau)}{1 + \mu \chi(x, \tau)},$$

and both

$$\phi(x, \dot{x}, \tau) \quad \text{and} \quad \chi(\dot{x}, \tau)$$

are given by the formulas (20), in which x and $\dot{x}$ are expressed in u and v according to the (24).

To find the values $u = a, v = b$, which are the first approximation for solving our equations, which is so-called "zero" solution, we must solve the following system of equations:

$$\int_0^{2\pi} f(a,b,\tau,0)\cos \tau d\tau = 0 \\ \int_0^{2\pi} f(a,b,\tau,0)\sin \tau d\tau = 0 \qquad (26)$$

Because of the (21) this system of equations is identical to

$$\int_0^{2\pi} \psi(a,b,\tau)\cos \tau d\tau = 0 \\ \int_0^{2\pi} \psi(a,b,\tau)\sin \tau d\tau = 0 \qquad (27)$$

In order to make the solution obtained this way stable, it is necessary to be

$$D_1(2\pi) + E_2(2\pi) < 0 \qquad (28)$$

and

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$$\begin{vmatrix} D_1(2\pi), & E_1(2\pi) \\ D_2(2\pi), & E_2(2\pi) \end{vmatrix} > 0 \ . \tag{29}$$

Here
$$\left. \begin{aligned} D_1(2\pi) \int_0^{2\pi} \left[\frac{\partial f}{\partial u} \right] \cos \tau d\tau, \quad E_1(2\pi) = \int_0^{2\pi} \left[\frac{\partial f}{\partial v} \right] \cos \tau d\tau \\ D_2(2\pi) \int_0^{2\pi} \left[\frac{\partial f}{\partial u} \right] \sin \tau d\tau, \quad E_2(2\pi) = \int_0^{2\pi} \left[\frac{\partial f}{\partial v} \right] \sin \tau d\tau \end{aligned} \right\} \tag{30}$$

and the symbols $\left[\dfrac{\partial f}{\partial u} \right]$ etc. mean that $\left[\dfrac{\partial f}{\partial u} \right]$ etc. are taken for $\mu = 0,\ u = a,\ v = b.$

As

$$\left[\frac{\partial f}{\partial u} \right] = \left[\frac{\frac{\partial \psi}{\partial u}(1 + \mu\chi) - \mu \frac{\partial \chi}{\partial u} \psi}{(1 + \mu\chi)^2} \right] = \left[\frac{\partial \psi}{\partial u} \right]$$

etc., and similarly

$$\left[\frac{\partial f}{\partial v} \right] = \left[\frac{\partial \psi}{\partial v} \right],$$

then the conditions of the equations (28) and (29) are reduced to

$$\int_0^{2\pi} \left[\frac{\partial \psi}{\partial u} \right] \cos \tau d\tau + \int_0^{2\pi} \left[\frac{\partial \psi}{\partial v} \right] \sin \tau d\tau < 0$$

(28₁)

$$\int_0^{2\pi} \left[\frac{\partial \psi}{\partial u} \right] \cos \tau d\tau \cdot \int_0^{2\pi} \left[\frac{\partial \psi}{\partial v} \right] \sin \tau d\tau - \int_0^{2\pi} \left[\frac{\partial \psi}{\partial u} \right] \sin \tau d\tau \cdot \int_0^{2\pi} \left[\frac{\partial \psi}{\partial v} \right] \cos \tau d\tau > 0 \tag{29₁}$$

Let us apply the recently reduced pattern of calculation to the considered particular cases. First consider the case of harmonic self-induction change.

Here:

$$\mu\psi(u,v,\tau) = -\left\{\left(\frac{m}{2}+2\vartheta\right)u+\left[\xi+\frac{\gamma_1}{4}(u^2+v^2)\right]v\right\}\cos\tau$$
$$+\left\{\left(\frac{m}{2}-2\vartheta\right)v+\left[\xi+\frac{\gamma_1}{4}(u^2+v^2)\right]u\right\}\sin\tau$$
$$+\frac{\beta_1}{2}(u^2-v^2)\sin 2\tau-\beta_1 uv\cos 2\tau$$
$$-\left[3\left(\frac{m}{2}+\gamma_1 uv\right)u-\gamma_1 v^3\right]\cos 3\tau-\left[3\left(\frac{m}{2}+\gamma_1 uv\right)v+\gamma_1 u^3\right]\sin 3\tau$$

(31)

and consequently the equations (20) take the following form:

$$\left(\frac{m}{2}+2\vartheta\right)a+\left(\xi+\frac{\gamma_1}{4}X^2\right)b=0$$

(32₁)

$$\left(\frac{m}{2}-2\vartheta\right)b+\left(\xi+\frac{\gamma_1}{4}X^2\right)a=0$$

(32₂)

where $X^2=a^2+b^2$ is a square of the amplitude of parametrically excited oscillations.

It follows from these equations that either

$$a = 0, b = 0,$$

(33)

or

$$\left(\xi+\frac{\gamma_1}{4}X^2\right)^2=\frac{m^2}{4}-4\vartheta^2.$$

(34)

To find out which of these values are physically feasible in these conditions let us consider the stability conditions (28) and (29).

Since in this case:

$$\left.\begin{array}{l} D_1(2\pi)=-\pi\left(\frac{m}{2}+2\vartheta+\frac{\gamma_1}{2}ab\right)\\[2mm] D_2(2\pi)=\pi\left(\xi+\frac{\gamma_1 X^2}{4}+\frac{\gamma_1}{2}a^2\right)\\[2mm] E_1(2\pi)=-\pi\left(\xi+\frac{\gamma_1 X^2}{4}+\frac{\gamma_1}{2}b^2\right)\\[2mm] E_2(2\pi)=-\pi\left(\frac{m}{2}-2\vartheta+\frac{\gamma_1}{2}ab\right), \end{array}\right\}$$

(35)

from (28) and (29) we have the following stability conditions. In case when a = 0, b = 0,

$$\vartheta > 0$$

(36₁)

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$$\frac{m^2}{4} - 4\vartheta^2 - \xi^2 < 0 \qquad (36_2)$$

and when $a \neq 0$, $b \neq 0$,

$$\vartheta > 0 \qquad (37_1)$$

$$\gamma_1 \left[mab - X^2 \left(\xi + \frac{\gamma_1}{4} X^2 \right) \right] > 0 \qquad (37_2)$$

The condition (36_1) or its identical condition (37_1) is always fulfilled, but it leads to the following from the conditions (36_2) and (37_2). First of all it follows from (36_2) that if only

$$\frac{m^2}{4} - 4\vartheta^2 \geq \xi^2 \qquad (36)$$

or if

$$\sqrt{\frac{m^2}{4} - 4\vartheta^2} \geq \xi \geq -\sqrt{\frac{m^2}{4} - 4\vartheta^2} \qquad (38)$$

then the resting condition of the oscillation system is unstable.

Thus the inequation (36) is a condition of oscillation excitation under periodic parameter change. If it is held, then a and b can not both be zero, and then the possible values of the stationary amplitude are obtained from the equation (34), i.e. given by the equation:

$$\frac{\gamma_1 X^2}{4} = -\xi \pm \sqrt{\frac{m^2}{4} - 4\vartheta^2} \quad . \qquad (34_1)$$

When the oscillation condition (36) is fulfilled the root is real and we have two possible values for X^2. The stability condition (37_2) gives us the answer to the question of choosing the root sign.

In fact, taking into account the equations (32_1) and (32_2) it is possible to write the latter condition as:

$$\gamma_1 X^2 \left(\xi + \frac{\gamma_1}{4} X^2 \right) > 0, \qquad (39)$$

which shows that the root sign in the formula (34_1) is equal to the sign γ_1.

Hence if $\gamma_1 < 0$ we have:

$$X^2 = \frac{4}{|\gamma_1|} \left(\xi + \sqrt{\frac{m^2}{4} - 4\vartheta^2} \right), \qquad (40_1)$$

whereas if $\gamma_1 > 0$ we have

$$X^2 = \frac{4}{\gamma_1} \left(-\xi + \sqrt{\frac{m^2}{4} - 4\vartheta^2} \right). \qquad (40_2)$$

Thus fulfilling the condition (36) and making a small adjustment of the system so that

$$\xi > -\sqrt{\frac{m^2}{4} - 4\vartheta^2} \text{ , if } \gamma_1 < 0 \qquad\qquad (41_1)$$

and
$$\xi < \sqrt{\frac{m^2}{4} - 4\vartheta^2} \text{ , if } \gamma_1 > 0 \qquad\qquad (41_2)$$

it is possible by means of periodic change of induction with frequency 2ω to excite oscillations of such frequency, the stationary amplitude of which will be given by the formulas (40_1) or (40_2), in the system that is approximately set to frequency ω.

As we can see from the formulas (41_1) and (41_2), the theory restricts the mismatch ξ at the first approximation only at one point, i.e. stable values of the amplitude outside the range of the values ξ that is determined by the oscillation excitation condition are possible too. In other words, the oscillations excited parametrically are "pulled". The obtained approximate formulas for the amplitude do not point the band that the considered "pulling area" covers. To answer one or another related question, we cannot confine ourselves to the considered "zero" approximation, it is necessary to take into account influence of the members that contain μ per amplitude of the fundamental harmonic as well as the role of overtones. It should be noted that the zero solution for the case similar results where the relationship between the flux and the current in the confining reactor choke is expressed by arkustangensoid ([19]), which is considered in the article of V.P. Gulyaev and V.V. Migulin, leads to the same results.

Let us consider the nature of the dependence of the parametrically excited oscillation amplitude on the magnitudes that define it. The Fig. 3 and 4 show the curves of X_2 dependence on the ξ mismatch, which can be named as the curves of the heteroparametric resonance. It is easy to see that these curves differ significantly both from the usual resonance curves and the 2nd type resonance curves.

As we can see from the Fig. 3, whilst
$$\xi < -\sqrt{\frac{m^2}{4} - 4\vartheta^2} \qquad\qquad (\text{if } \gamma_1 < 0)$$

there are no noticeable oscillations in the system.

When:
$$\xi_1 = -\sqrt{\frac{m^2}{4} - 4\vartheta^2}$$

parametric oscillations occur starting with very small amplitudes and increase with further increase of ξ. As this takes place, X_2 increases rectilinearly until at some value

$$\xi_2' > \sqrt{\frac{m^2}{4} - 4\vartheta^2}$$

oscillations abruptly die out. When we have a reverse mismatch, oscillations occur even if

$\xi_2 = \sqrt{\dfrac{m^2}{4} - 4\vartheta^2}$ and then

with further decrease of ξ they reduce until X_2 is zero again if

$\xi_1 = -\sqrt{\dfrac{m^2}{4} - 4\vartheta^2}$.

Hence there is a "pulling loop" only at one point (Fig. 3).
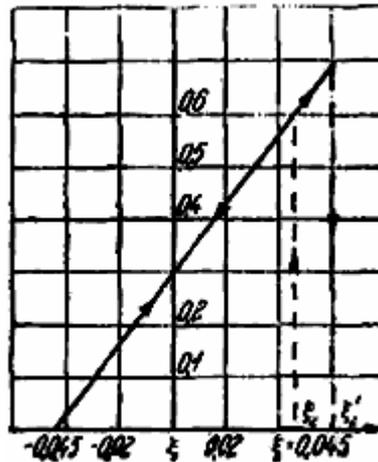
As we can see from

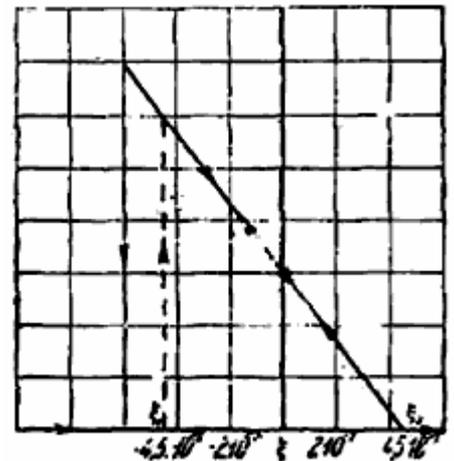**Fig. 3.** Heteroparametric resonance curve ($\gamma_1 < 0$).

Fig. 4. Heteroparametric resonance curve ($\gamma_1 > 0$).

the Fig. 4, if $\gamma_1 > 0$ we have a reversed situation: X_2 increases with reducing of ξ and there is a pulling loop when $\xi = \xi.^1$ The maximum magnitude of X_2 within the oscillation excitation area is:

$$X_{max}^2 = \frac{8\sqrt{\dfrac{m^2}{4} - 4\vartheta^2}}{|\gamma_1|},$$

i.e. the less γ_1 is, the more the maximum magnitude is.

We obtain quite similar results in case of a harmonic change in capacity. In fact in this case:

$$\mu\psi(u, v, \tau) = \left\{ \left[\frac{m}{2}(1-\xi) - 2\vartheta \right] v + \left(\xi + \frac{\gamma_1}{4} X^2 \right) u \right\} \sin\tau$$
$$- \left\{ \left[\frac{m}{2}(1-\xi) + 2\vartheta \right] u + \left(\xi + \frac{\gamma_1}{4} X^2 \right) v \right\} \cos\tau + \tag{42}$$

+ members that contain the highest harmonics.

Comparing the members that contain $\sin\tau$ and $\cos\tau$ with the corresponding members of the formula (31), we can see that they are derived from the latter by means of replacing m by $m(1 - \xi)$. Hence all the conclusions that were drawn in considering the problem with periodically changing self-induction can be directly extended to the case of capacity change.

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[1] As experiments show (see the article by V.A. Lazarev) both of these cases can be realized.

In particular, in case of capacity the boundaries of the parametric excitation area can be expressed in:

$$\frac{m^2}{4}(1-\xi)^2 - 4\vartheta^2 > \xi^2,\tag{43}$$

whence

$$-\frac{m^2}{4} + \sqrt{\frac{m^2}{4} - 4\vartheta^2} > \xi > -\frac{m^2}{4} - \sqrt{\frac{m^2}{4} - 4\vartheta^2}\tag{44}$$

that is identical to the formula (38) up to $\dfrac{m^2}{4}$.

Up till now we have been considering the phenomenon of oscillation excitation by means of periodic parameter change in oscillation system without regeneration. Having parametric impact to the regenerative system we face a number of noteworthy features, therefore it is necessary to consider this case in more detail in the next section.

§ 4. Parameter change in regenerative system

Since in this case substituting (24) into (20$_2$) we have:

$$\mu\psi(u, v, \tau) = \left[\left(k - \tfrac{m}{2} + \tfrac{\gamma_1}{4}X^2\right)u - \xi v\right]\cos\tau$$
$$+ \left[\left(k + \tfrac{m}{2} + \tfrac{\gamma_1}{4}X^2\right)v + \xi u\right]\sin\tau \tag{45}$$

+ members with higher harmonics

the equations (27$_1$) and (27$_2$) for a and b take the following form:

$$\left.\begin{array}{l} \left(k - \dfrac{m}{2} + \dfrac{1}{4}\gamma_1 X^2\right)a = \xi\cdot b \\[2mm] \left(k + \dfrac{m}{2} + \dfrac{1}{4}\gamma_1 X^2\right)b = -\xi\cdot a, \end{array}\right\}\tag{46}$$

whence we have either a = 0, b = 0

or
$$\left(k + \frac{1}{4}\gamma_1 X^2\right)^2 - \frac{m^2}{4} = -\xi^2\tag{47}$$

or
$$\frac{1}{4}\gamma_1 X^2 = -k \pm \sqrt{\frac{m^2}{4} - \xi^2}\tag{47$_1$}$$

To determine the physical conditions that are essential for existence of a particular solution we turn to the stability conditions.

Since in this case:

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$$\left.\begin{array}{l} \dfrac{1}{\pi} D_1(2\pi) = k - \dfrac{m}{2} + \dfrac{1}{2}\gamma_1 a^2 + \dfrac{1}{4}\gamma_1 X^2 \\[3mm] \dfrac{1}{\pi} D_2(2\pi) = \xi + \dfrac{1}{2}\gamma_1 ab \\[3mm] \dfrac{1}{\pi} E_1(2\pi) = -\xi + \dfrac{1}{2}\gamma_1 ab \\[3mm] \dfrac{1}{\pi} E_2(2\pi) = k + \dfrac{m}{2} + \dfrac{1}{2}\gamma_1 b^2 + \dfrac{1}{4}\gamma_1 X^2, \end{array}\right\} \tag{48}$$

then the conditions (28) and (29) take the following form:

For the case a = 0, b = 0

$$k < 0 \tag{49_1}$$

$$k^2 - \frac{m^2}{4} + \xi^2 > 0 \tag{49_2}$$

For the case a ≠ 0, b ≠ 0

$$R + \frac{1}{4}\gamma_1 X^2 < 0 \tag{50_1}$$

$$\gamma_1 R X^2 > 0 \tag{50_2}$$

Here R means $\pm\sqrt{\dfrac{m^2}{4} - \xi^2}$.

It is possible to draw the following conclusions from these formulas. First of all from (49_1) and (49_2) it follows that when k < 0, i.e. when the system is not self-excited (cf. the formula (36)), parametric excitation is possible only if

$$\frac{m^2}{4} - k^2 > \xi^2 \tag{51}$$

Comparing this with the formula (36) for an over generated system we can see that we have here a smaller magnitude instead of 2ϑ . Hence due to use of regeneration it is also possible to excite oscillations in the system parametrically even if the specified parameter modulation depth m is not sufficient to satisfy the condition (36). This conclusion formed the basis of some of the experiments described below.

If the condition (51) is fulfilled, the system is unstable with $\alpha = 0$, b = 0. In case of stationary periodic motion it is expressed by the formula (47_1). It follows from the conditions (50_1) and (50_2) that the condition expressed by them is stable only if both $\gamma_1 < 0$ and $R < 0$ at a time. Thus we conclude that the amplitude of the stationary periodic oscillations is expressed by:

$$X^2 = \frac{4}{|\gamma_1|}\left[k + \sqrt{\frac{m^2}{4} - \xi^2} \right] \tag{47_2}$$

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This formula is right both for $k < 0$ (non-self-excited system) and for $k > 0$ (self-excited system). Let us consider the first case. First, note that the condition of the X realness coincides with the condition of parametric oscillation excitation (51). It follows thence that the phenomenon of "pulling" is missing both under autoparametric excitation and "soft" mode of excitation.

If we compare the formula (47_2) with an appropriate formula for the amplitude of the oscillations excited autoparametrically: ([18])

$$X^2 = \frac{4}{|\gamma_1|} \left[k + \sqrt{\frac{\lambda_0^2}{9} - \frac{\xi^2}{\beta^2}} - \gamma_1 \frac{\lambda_0^2}{18} \right] \tag{52}$$

we will see that both formulas are quite similar and almost the same with small λ_0. Hence the curves of the heteroparametric resonance in this case are quite similar to the curves of the autoparametric resonance considered earlier ([17]) and ([18]) (resonance of the 2nd type), moreover the external force is replaced here by the modulation depth m. The Fig. 5 shows the heteroparametric resonance curve found by the formula (47_2). As we can see here the resonance curve with the amplitude limited by nonlinear resistance differs significantly from heteroparametric resonance curve with the amplitude limited by nonlinear self-induction (cf. Fig. 3 and 4).

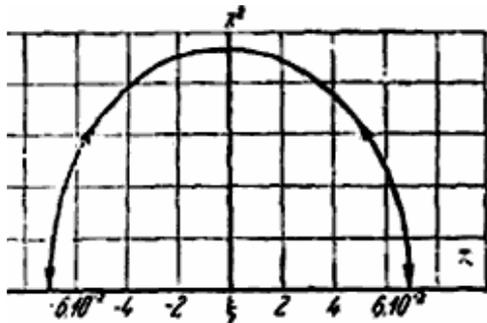When a parameter in the self-excited system ($k > 0$) is changed we can draw the following conclusions. First of all, the fact of the stable periodic solution existence (47_2) points that under the heteroparametric influence upon the self-oscillating system the phenomenon of forced synchronization occurs ("frequency dragging"). Now since the realness of X is dictated only by the realness of the root when $k > 0$, we have the following formula for the

Fig. 5. Heteroparametric resonance curve in the regenerative system (theoretical).

"dragging area":

$$\frac{m}{2} \geq \xi \geq -\frac{m}{2} \tag{53}$$

which implies that this area is larger than the excitation area in non-self-excited system (51). Note that the self-oscillations decrease significantly on both sides of the dragging area, where there is no periodic process, and with sufficiently large influence amplitude they are completely "damped". Approximate theory of this phenomenon that is similar to the phenomenon of asynchronous damping will be given elsewhere.

EXPERIMENTAL PART

The following experiment was made to check the possibility of oscillation excitation in an oscillatory system by means of a single periodic change of its parameters without introducing any emf into it. As was shown above, such stimulation can be expected only if the condition

$$m > \frac{2}{\pi}\varepsilon \qquad (*)$$

where m is the relative magnitude of the parameter (its so-called modulation depth) and ε is the average logarithmic decrement of the system. Thus it was necessary to realize, on one hand, a very efficient way to change the parameter and, on the other hand, the system probably with a smaller ε. Since further the maximum power of parametrically excited oscillations is equal to

$$W = \frac{m}{4}\omega CV^2,$$

then to get any sensible power with feasible frequency (2ω) of the parameter change, it is necessary to have capacity C of considerable value in the circuit that is able to withstand high voltage. Due to the complexity to fulfill the variable capacity of a required value that assumes an adequate modulation depth with wanted high frequencies in laboratory conditions, we refused to change the capacity and chose self-induction as a periodically varied parameter. Out of the various ways of the periodic self-induction change, at first we stopped at the following for various reasons. If we introduced any conductive body into the variable field of the self-inductor L (in the simplest case it is a shading coil), then, as we know, the magnetic field energy and consequently the effective L would decrease in view of Foucault current induced in the body. On this basis we applied the following (Fig. 6, 7 and 8) as a widget that allowed changing the effective magnitude of the self-induction periodically that occurred
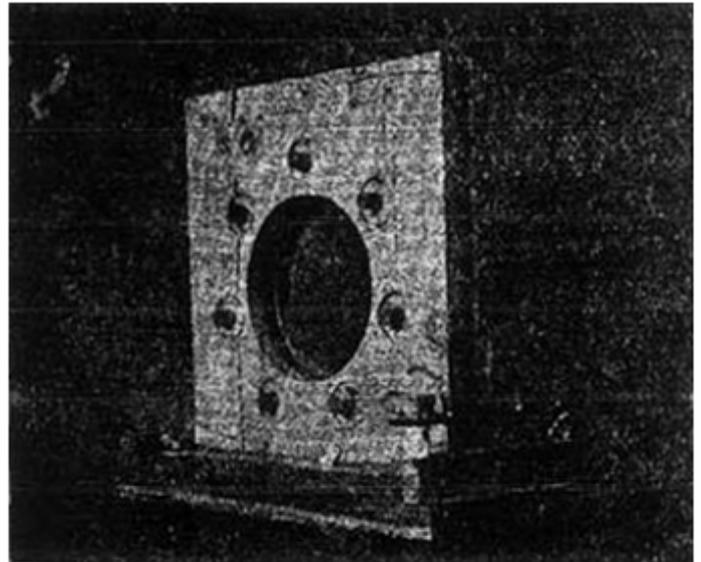

Fig. 6.
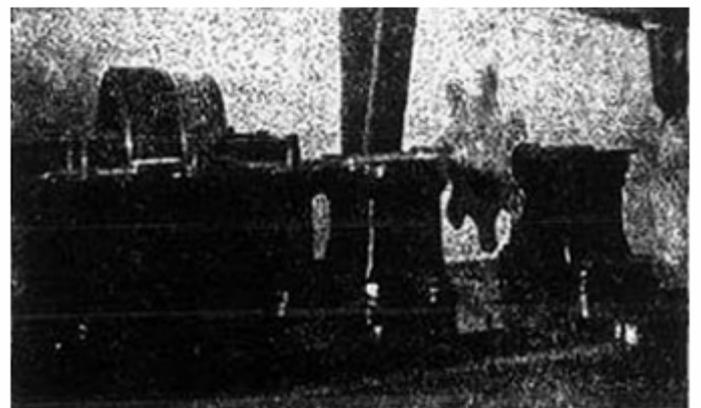
Fig. 7.

conveniently and with required frequency. The variable induction here consisted of two groups of slab coils (7 in each) (Fig. 6) mounted on two parallel peripheral discs on two parallel circles, so that there was narrow space like a split between the coil sides facing each other. A metal rotating disk with peripheral teeth-like notches (7 like the number of coils) (Fig. 7) was placed inside of the split. The notches were placed in such a way that their middles coincided with the centers of the coils at some time during the rotation. Thus the periodic change of self-induction was achieved here by the fact that when you rotated the disk, the teeth got in and out of the field coils alternately. In the first case the effective self-induction obviously would be minimal and in the second - the maximum. Since such a disk (for example, of duralumin) could be rotated at very high rates (the peripheral speed was about 220 m/sec in our experiments), consequently, using the said way of induction change it was possible to perform high frequencies (1700-2000 per second) of parameter change and to obtain oscillations of sufficient power. Note that to increase the self-induction, as well as for greater field concentration in the space between the coils, they were provided with cores of divided iron.

In our first experiments with oscillation excitation in the system with periodical change of self-induction in early 1931 to fulfill the excitation conditions we used the principle of regeneration by means of a vacuum valve for the excitation conditions, since the first made coil system has too much resistance, and the logarithmic decrement of the system ε was significantly greater than 0.12, whereas the measured (from the definition of the proper frequency of the system with two extreme positions of the disk (teeth in the field of coils and teeth outside the field of coils)) depth of self-induction modulation was only 0.07, i.e. m was smaller than $\frac{2}{\pi}\varepsilon$.
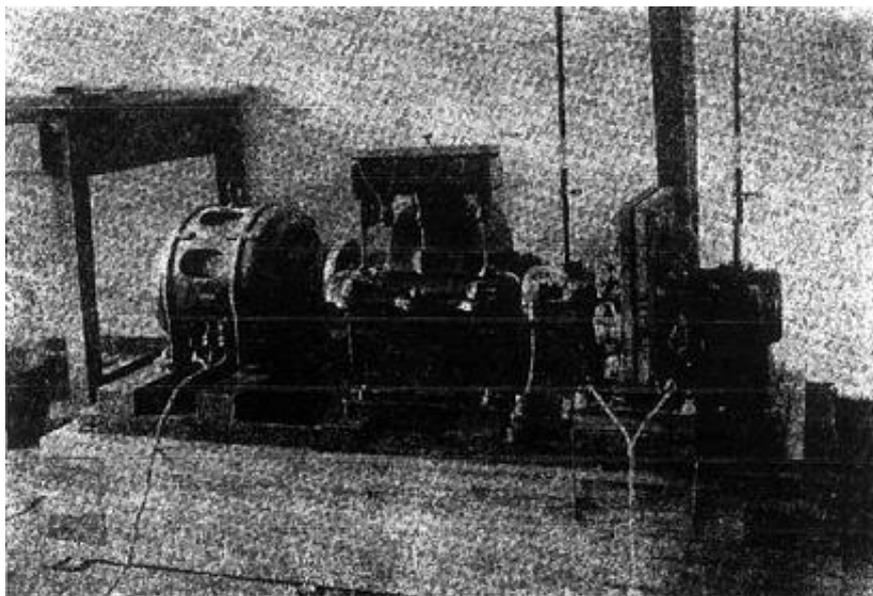

Fig.8.

The regeneration scheme with parallel feed shown in Fig. 9 was chosen to avoid any obvious current and voltage in the oscillating loop in the initial state.

Here the return coupling was performed through the capacity C_n, the change of which allowed us to adjust smoothly the value of coupling. Here is the scheme data. The oscillating loop consisted of the mechanically changeable self-induction L_1 described above and the additional self-induction $L_2 = 0.1$ henry and Hartmann & Braun variable inductor that allows the variation from 11.3 to 16.5^{-2} H served for a coarse adjustment. The loop capacity consisted of the constant part C_1 of 70,000 cm and the variable capacitor C_2 (the maximum capacity is 11,200 cm) connected to it in a parallel way for a fine adjustment. The total ohmic resistance of the loop was about 90 ohms, not counting losses. The distance between the coils (split width) was 5 mm and thickness of the duralumin disk was 3 mm. The vacuum valve was Micro (*translator's note*: Micro is the name of vacuum valves made in the USSR in 1920-1930). Anode voltage was 240 V. The disc was put on the axle, which was rotated by an engine with the reducing gear (1:10) of the high frequency machines` type, by V.P. Vologdin. With the number of engine turns that was equal to 1400-1500 turns/min (the number of disk turns is 10 times greater) we obtained the frequency of n self-induction change from 1630 to 1750 per second having the disk of 7 teeth.

The experiment was held in the following way. At first, having a fixed disk (or with rotation, the speed of which is not sufficient for oscillation excitation) the lamp's regime was chosen, in order that with sufficient return coupling (adjusted by C_n) and small adjustment of the system to the half frequency of the parameter change soft self-oscillation excitation was obtained, if possible, and then the coupling would decrease so much that self-oscillations would nor occur within the entire area of the adjustment. After that the disk was set in motion. When it reached full speed, the oscillations, frequency of which was twice as little as frequency of self-induction change, occurred in the system.
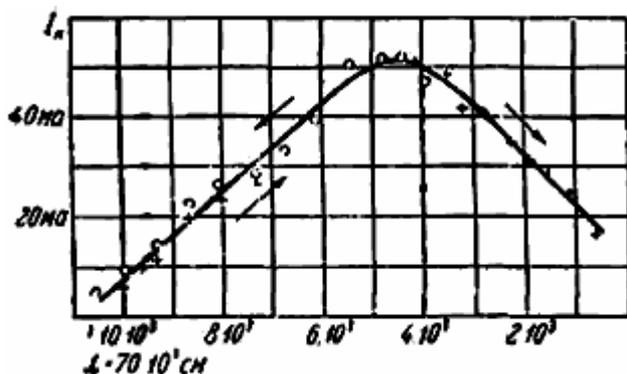
With a smooth change of loop capacity (i.e. proper frequency of the system) the oscillation frequency remained unchanged up to a certain detuning, after which the oscillations ceased. In fact, it was heteroparametric oscillation excitation that took place here and not half frequency oscillation excited in a regenerative system under the influence of variable current
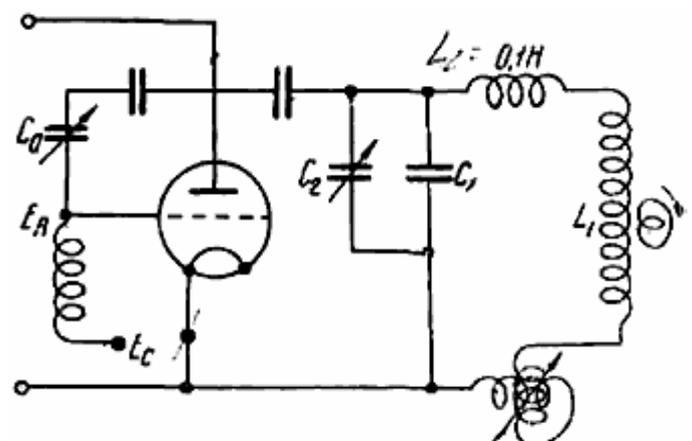
Fig. 9. Parametric excitation scheme in the system with regeneration.

pulses, which were induced by any field (like earth's magnetic field) in the teeth during rotation of the disk and created emf of the parameter change frequency (case of resonance of the 2nd type), that

is all clear from the following. For instance, if the maximum current in the loop with self-excitation was equal to only 9 mA having a constant component of the anode current i_a, which was equal to 1.4 mA, then with heteroparametric excitation it would reach 40mA, when i_a = 1,8 mA. Hence the loop was supplied with power by the disk due to the parameter change and not by the battery that fed the lamp as in the case of autoparametric excitation.

Fig. 10. Heteroparametric excitation scheme in the system with regeneration (experemental).

Fig. 10 shows the curve of the relations between the amplitude of the oscillations that occur with the parameter change and the oscillatory system detuning. Since in this case C_a = 44° and the self-oscillation occurred only in the range from C_a = 77 ° to C_a = 93°, then there was no self-excitation in the entire parametric excitation area: when the engine was stopped or rotation speed of the disk changed beyond the range for parametric excitation, the oscillations ceased in the loop. The oscillation frequency remained constant and exactly equal to a half of the parameter change frequency (n) over the whole curve of the parametric resonance (7 multiplied by the number of the disk turns per second). The measurements were made aurally by means of Siemens & Halske frequency meter.

Except for the disk made of duralumin we also made experiments with a disk of iron of the same shape, but with thickness equal to 2 mm. Despite the fact that the stator coils were pulled together to the distance of 4 mm in order to enhance the field concentration, there was no parametric excitation effect. The control measurement of the self-induction modulation depth showed that, as might be expected, the iron disk, acting as iron towards increasing of L, on one hand, and as the metal towards decreasing, on the other hand, caused a much smaller change in induction, making at a time big losses in the system.
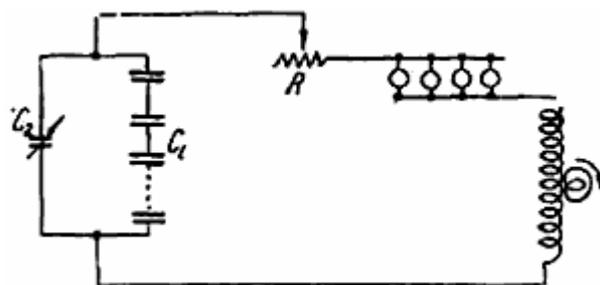
Fig. 11. Scheme of parametric excitation in the system without recycling.

Once the heteroparametric excitation effect in the regenerative system was specified, we began experimenting with the system without regeneration. For this purpose the stator coils were changed, the core of transformer iron was expanded (diameter - 2.2 cm, length - 6.5 cm) and the diameter of the coil block wire was

increased (0.9 mm). Due to these measures we managed, on one hand, to increase the field concentration of coils and thus increase the self-induction modulation depth (14.5%), and, on the other hand, to reduce ohmic losses in the loop (the resistance of the stator coils was reduced from 84.5 to 21 ohms. Since here $\varepsilon \sim 0.14$, so the condition (*) was satisfied and it was possible to expect the parametric oscillation excitation in this system without regeneration. In fact, when adjusting the oscillatory system (made by the scheme of Fig. 11, in which there are no obvious sources of current or voltage) to a frequency equal or similar to half frequency of the system self-induction change by means of the capacitor C_2, powerful oscillations with frequency equal to a half of the self-induction change frequency appeared in it. The oscillation amplitude here increased rapidly until the insulation, loop capacitor or leads fault occurred. In our experiments the voltage used to reach 12-15,000 V. In order to obtain stationary regime, it was necessary to introduce a conductor with nonlinear characteristic into the system in accordance to the theory. In our first experiments a number of incandescent bulbs (100 watt), which could be switched on in the loop in a parallel way, were taken as a conductor (Fig. 11). Here are the schema data. The loop capacity consisted of 17-20 series-connected capacitors (2*F each) and oil capacitor of variable capacity C_2 (11,000 cm), which was series-connected to constant capacity of 3,000 cm. The maximum and minimum values of self-induction of the stator coils were

$$L_{max} = 0,229 \text{ H}, \qquad\qquad L_{min} = 0,193 \text{ H}.$$

The lamp resistor mentioned above served as a load and the resistor R was taken for a smooth adjustment of the resistance introduced into the system. Coarse adjustment was made by changing the number of series-connected capacitors and more soft adjustment was made by means of the oil capacitor. It should be noted that due to the large instability of main voltage, which fed the engine, the number of the disk's turns was much changed and it was necessary to make frequent adjustment, as the variable capacitor allowed adjusting the loop frequency only within fairly narrow limits. This fact greatly complicated the experiments and gave us no opportunity to make all measurements in this scheme.

We mention the following out of all the experiments that were made. First of all, it should be pointed that the introduction of incandescent bulbs actually makes it possible to obtain and regulate the stationary oscillation amplitude in wide range (up to 5 A, since the engine power and the wire section of the coils did not allow a greater load). However, the filaments` thermal inertia sometimes causes peculiar phenomenon of "swaying" the amplitude, which lies in the fact that the amplitude increases wavily and not gradually: the bulbs burn now stronger, now weaker. Sometimes this

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[1]It should to be noted that designing, making and setting up of this and depicted in the following devices was performed with the significant participation of mechanical designer M. I. Rzyankin.

phenomenon, which is often associated with great overvoltage, lasts for several minutes. It is possible to avoid it notably by means of choosing a proper regime of the system. Also it should be noted that in the following experiments that were made in the present year and were depicted in detail by V. A. Lazarev in the following article, we applied a more convenient and technically perfect way to adjust a stationary amplitude magnitude based on the use of a nonlinear relationship between magnetic flux and current in a special choke introduced into the oscillating loop circuit.

Here are some results of the measurements performed with incandescent bulbs as a load. The dependence of voltage on the capacitor on the resistance magnitude introduced into the system is shown in Fig. 12. As we can see here the voltage decreases gradually, while the load increases. The oscillation cease when the 28 ohms resistance is introduced. If we assume this value as an extremum and take into account all the other losses in the system, like the coil block resistance, losses in the duralumin disk, in iron, dielectric losses in capacitors, then the total logarithmic
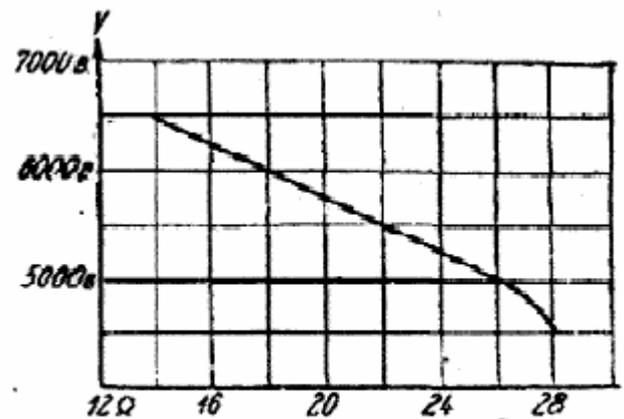
Fig. 12. Dependence of voltage amplitude at capacitor on resistance.

decrement ε of the proper oscillations of the system is about 0.20. Since the self-induction modulation depth measured in these conditions is 0.14, the ratio $m > \dfrac{2}{\pi}\varepsilon$, which is the excitation condition, is still being fulfilled.

The following, more detailed experiments were performed with a different experimental plant, in which the system of stator coils was changed in order to increase the modulation depth (40%) and power (up to 4kV). These coils of thinner wire were wound around almost closed cores of divided iron. The experiments performed with this plant that confirmed both the qualitative and quantitative conclusions of the theory are described in detail the article by V. A. Lazarev placed below. We only note that except for the duralumin disk we applied also a copper disk in this plant, which gave approximately the same results.

We have already reported in this journal on the experiments with the oscillation excitation by means of periodic change of the system capacity, also in accordance to the theory.

In conclusion, we consider it necessary to express our sincere gratitude to I. M. Borushko and V. A. Lazarev, who took a significant part in the experiments described above.

ON ELECTRIC OSCILLATION EXCITATION BY MEANS OF PARAMETRIC CHANGE
by L. Mandelstam and N. Papalexi

In this work the theory of electric oscillation excitation in oscillatory system by means of periodic parameter change with no applied electromotive forces is given. This theory, which is based on general methods of differential equation periodic solution stated by Henri Poincare, will be applied to the special cases of parameter change. These are the cases of oscillation excitation (excitation conditions, stationary amplitude, etc.) that can be observed both with a sinusoidal self-induction or capacity change in a nonlinear oscillatory loop (that contains an iron choke) or in a sinusoidal self-induction change in series-connected vacuum valves. The experimental part contains the description of the attempts to excite oscillations by means of periodic change in self-induction or capacity in accordance with the theory.

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