ANGULAR MOMENTUM, PARAMETRIC OSCILLATOR AND OVER UNITY

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ABSTRACT

The goal of this work is to present mathematical proof that the law of conservation of energy is not valid in part of a system where the law of conservation of angular momentum is valid. Also, possibility of getting energy surplus or over-unity energy by using pendulum as parametric oscillator will be discussed.

In this work the following will be discuss:

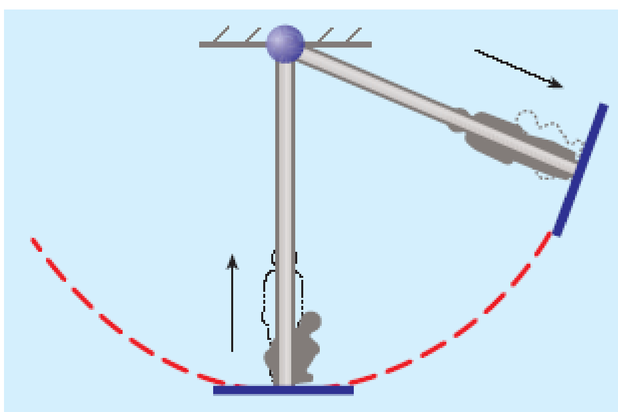
- the law of conservation of angular momentum,
- angular momentum and conflict with the law of conservation of total energy in orbits of central forces (gravitational, electrostatic, etc.),
- angular momentum and corruption of centrifugal force,
- an experiment of increasing potential and kinetic energy in the same time when the law of conservation of angular momentum was supposed to be valid,
- possibility of getting energy surplus out of pendulum which works as parametric oscillator,
- importance of egg shaped forms and angular momentum for fluids.

Key words: angular momentum, over-unity, parametric oscillator, egg shape, implosion.

INTRODUCTION

Soon after publishing document *Theory of Gravity Machines* [1] author constructed new wooden model of Veljko Milkovic's two stage mechanical oscillator to test some ideas for control of movable pivot point of the pendulum as explained in above mentioned work. Neodymium super magnet was used to lock lever arm with the mass in its upper position to create some lag, but there were no significant improvement concerning the time of pendulum swing. The same happened with idea of horizontal movement of the pivot point. Author also tested several simple ideas for construction of Bessler's wheel, but none of them worked. This was the reason for conclusion that Bessler's wheel was probably a fraud.

Latter, author decided to perform search for Bessler wheel on internet, for the last time, to check some ideas other people had about it. Most interesting were ideas of John Collins on his site [2] and especially idea of pumping the swing by standing and squatting method which was named as parametric oscillator. That method as well method of leaning, called driven oscillator, was mathematically described by Tareq Ahmed Mokhiemer in his document *How to pump a swing* [3]. Author read the document again and noticed that Mr. Mokhiemer got conflicting results between energy pumped into the system and energy passed by a child to the swing, for method of parametric oscillator like on *picture 1* bellow.



Picture 1

Mathematics in document was pretty complex and Mr. Mokhiemer didn't make any comment about discrepancy for this method except that it was only conflicting result in his paper.

Author decided to investigate idea of the swing as parametric oscillator further. The result of his investigation is given in this work.

After publishing first version of this work, author has received critics on winding pendulum and ideal parametric oscillator which didn't exist in practice, because the distance can not be shorted instantaneously. Author has updated the work with appendix A in which real situation has been described, where centrifugal force and pendulum velocity kept changing together with change of the distance of pendulum bob. Appendix had many formulas and one wrong conclusion for the case of extended pendulum handle. In this version, the error has been corrected, section with parametric oscillator has been redone and moved at the end of work and some new comments and chapters were added.

ANGULAR MOMENTUM

In mechanics there are two basic measurements of motion of a body with mass *m* and velocity *v*. When movement transforms in another form like potential energy or heat the measurement of motion is kinetic energy *E* and its formula is:

$$E = \tfrac{1}{2} m v^2 \quad (1)$$

For passing the movement from one body to another it is important to know momentum of the body. In some countries it is called quantity of the motion as Rene Descartes named it. It is second measurement of the motion and its formula is below:

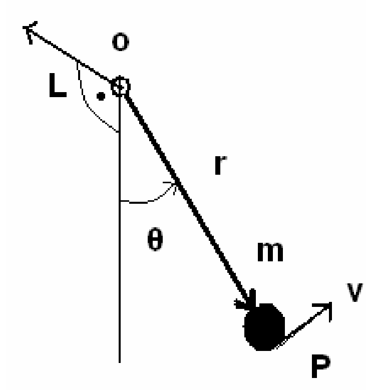
$$\mathbf{P} = m \mathbf{v} \quad (2)$$

Kinetic energy *E* is scalar which means it has only intensity, but momentum $\mathbf{P}$ is vector which has direction same as its velocity $\mathbf{v}$. If external force doesn't act on the body its momentum will stay the same. It is the law of conservation of momentum.

For a body rotating around an axis angular momentum $\mathbf{L}$ has been defined as a vector equal to cross product of position vector of a body $\mathbf{r}$ and linear momentum of a body, as down:

$$\mathbf{L} = \mathbf{r} \times \mathbf{P} = \mathbf{r} \times m \mathbf{v} \quad (3)$$

Note that formula (3) is used for mass with small volume and for a particle. For large rotating bodies moment of inertia is used for both, angular momentum and kinetic energy. In this work, body will be regarded as particle which means that its mass has small size in comparison with distance of rotating axis or pivot point *o*. It means that position vector $\mathbf{r}$ has intensity (length) several times greater than diameter of a body, see bellow.



Picture 2

Vector ***L*** is normal to the plane of rotation and its intensity can be found by formula bellow:

$$L = r\ m\ v\ \sin(\theta) \tag{4}$$

In all cases in this work, angle between position vector ***r*** and velocity ***v*** is 90 degrees because velocity is tangential to circular path of the movement, so above formula becomes:

$$L = r\ m\ v \tag{5}$$

The time derivative of angular momentum (3) is called torque. It is given bellow:

$$\frac{\overrightarrow{dL}}{dt} = \frac{\overrightarrow{dr}}{dt}\ x\ m\ \vec{v} + \vec{r}\ x\ m\ \frac{\overrightarrow{dv}}{dt}$$

$$\frac{\overrightarrow{dL}}{dt} = \vec{v}\ x\ m\ \vec{v} + \vec{r}\ x\ m\ \vec{a}$$

$$\frac{\overrightarrow{dL}}{dt} = 0 + \vec{r}\ x\ \vec{F}$$

$$\frac{\overrightarrow{dL}}{dt} = \vec{M} \tag{6}$$

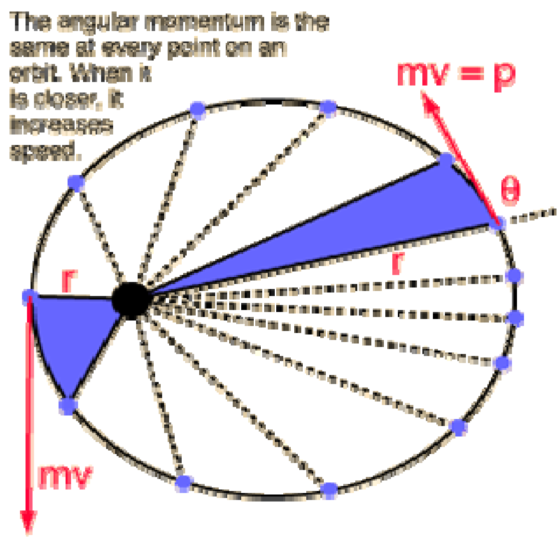
Torque ***M*** is equal to cross product of position vector ***r*** and external force ***F***.

The Law of Conservation of Angular Momentum of a Particle

If there are no acting external forces on the body then torque is zero and time derivation of the angular momentum (6) becomes:

$$\frac{d\overrightarrow{L}}{dt} = 0 \Rightarrow \vec{L} = const \tag{7}$$

It also means that angular momentum ***L*** will stay constant all the time. Such cases are in orbiting planets around the Sun. Gravitational force of the Sun acts from the center of the planet towards the center of the Sun (axis of the rotation) and can not create torque. By looking back into formula (5) it can be seen that any change in distance ***r*** would cause the change in velocity ***v*** because mass *m* is never changed. Increase of the distance ***r*** would proportionally decrease velocity ***v*** and vice versa, see example on *picture 3* bellow.



Picture 3

ENERGY BALANCE FOR CENTRAL FORCES

Central forces are such forces where direction of its action is passing trough center of two bodies. Examples are gravitation force between the Sun and planets or a planet and satellite and also electrostatic force in atom of Hydrogen. In all above cases a body with smaller mass is rotating around the body with bigger mass along circular or elliptical orbit. Because only external force is central force and it doesn't create moment of force ***M*** or torque, the law of the conservation of the angular momentum ***L*** is valid here.

Central forces are centripetal forces because they tend to move a body inward towards the center of the Sun or planet along radial path.

Centrifugal Force

Centrifugal force is called fictitious force because it is a reaction on centripetal force and accts in opposite direction of centripetal force. Its origin is in inertia of the body and first Newton law which says that a body tends to continue motion in the same direction if no external force acts on it. If a centripetal force acts on a body its inertia will be felt as centrifugal force. The example is a driver sitting in a car which started to make turn to the right. The body of the driver wants to continue motion in the straight line and will feel pressure of the left door as centripetal force. The door will feel pressure of driver's body as centrifugal force.

This force is normally not included in differential equation which describe trajectory of the body because mass of the body is included as measure of its inertia. However, formula for centrifugal force is frequently used in physics independently when necessary. The same logic will be used in this work. Down is formula for centrifugal force:

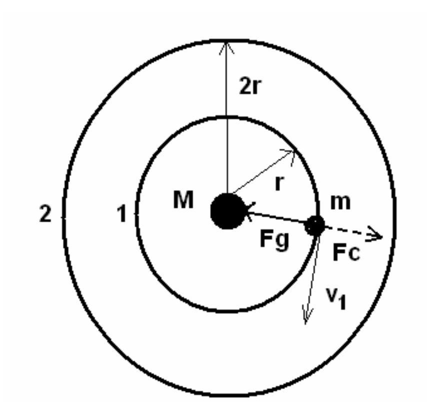
$$F_c = \frac{mv^2}{r} \tag{8}$$

<u>Here *r* is radius of curvature and it is the same as the distance from the center only in the case of circular paths</u>.

It is necessary to note that if centripetal force were cancelled than inertial mass would continue to move in straight line and will have kinetic energy it had in the moment centripetal force was cancelled.

Total Energy in Two Orbits

Down on *picture 4* is the Earth with mass *M* and a satellite with mass *m* in circular orbit 1. Here, total energies of orbit 1 and orbit 2 will be calculated.



Picture 4

Total energy consists of potential and kinetic energies. The satellite is circulating along orbit 1 with tangential velocity v_1 and energy spent to raise it to orbit 2 is equal to potential energy increase of orbit 2, ΔE_p.

Formula for *g*ravitational force ***Fg*** is given bellow:

$$\overrightarrow{F_g} = -G \frac{M\ m}{r^2} \overrightarrow{r_0} \tag{9}$$

where *G* is universal gravitational constant and vector $\boldsymbol{r_0}$ has intensity of one and serves to point direction only. Because the force ***Fg*** has opposite direction of vector $\boldsymbol{r_0}$ there is minus sign in above formula. Variable *M* is mass of the Earth and *m* is mass of the satellite.

Gravitational force ***Fg*** must be in balance with centrifugal force ***Fc,*** otherwise the satellite would leave the orbit. It means that *Fc* = *Fg*, so:

$$\frac{mv_1^2}{r} = \frac{GMm}{r^2} \Rightarrow v_1^2 = \frac{GM}{r} \tag{10}$$

Kinetic energy in orbit 1 can be found by using formula (1) as:

$$E_{k1} = \frac{1}{2} m v_1^2 \tag{11}$$

Potential energy is defined as reserve of the work and has opposite sign of work done by gravitational force to move satellite from point 1 to point 2. Point 2 is point of reference and it is sometimes infinity and sometimes the surface of the planet. In this case it is logical that point of reference is surface of the Earth with radius *R*. Work done by gravitational force to raise satellite from surface of the Earth to orbit 1 is:

$$A = \int \overrightarrow{F}\,\overrightarrow{dr} = -GMm \int_R^r \frac{dr}{r^2} = GMm\left(\frac{1}{r} - \frac{1}{R}\right) \tag{12}$$

Potential energy in orbit 1 is equal to:

$$E_{p1} = -A = G\ M\ m \left(\frac{1}{R} - \frac{1}{r} \right) \tag{13}$$

Potential energy in orbit 2 is equal to:

$$E_{p2} = G\ M\ m \left(\frac{1}{R} - \frac{1}{2r} \right) \tag{14}$$

Increase of potential energy in orbit 2 is equal to energy spent in order to overcome gravitational force from orbit 1 to orbit 2 and is equal to:

$$\Delta E_p = E_{p2} - E_{p1} = \frac{G M m}{2r} \tag{15}$$

By changing (10) into (15) potential energy increase in orbit 2 is:

$$\Delta E_p = \frac{m v_1^2}{2} = E_{k1} \tag{16}$$

To find velocity in orbit 2, the law of conservation of angular momentum ***L*** will be used. Formula (5) is valid for both orbits, so:

$$L_1 = L_2$$

$$r\, m\, v_1 = 2r\, m\, v_2$$

$$v_2 = \tfrac{1}{2} v_1 \tag{17}$$

Kinetic energy in orbit 2 is:

$$E_{k2} = \frac{1}{2} m v_2^2 = \frac{1}{2} m \left(\frac{1}{2} v_1\right)^2 = \frac{1}{4} E_{k1} \tag{18}$$

Total energy in orbit 1 is equal to sum of potential and kinetic energies:

$$T_1 = E_{p1} + E_{k1} \tag{19}$$

Total energy in orbit 2 is equal to:

$$T_2 = E_{p2} + E_{k2} = (E_{p1} + \Delta E_p) + E_{k2} \tag{20}$$

By changing (16) and (18) into (20) it becomes:

$$T_2 = (E_{p1} + E_{k1}) + E_{k1} / 4 = E_{p1} + 5/4\, E_{k1} \tag{21}$$

Total energy difference is:

$$T_2 - T_1 = 1/4\, E_{k1} \tag{22}$$

Because energy invested to move the satellite from orbit 1 to orbit 2 was equal to ΔE_p and according to formula (16) it is equal to E_{k1}, for the same amount should total energy of orbit 2 be greater than total energy of orbit 1. However, according to formula (22) that amount is smaller, which means that there is energy loss of orbit 2 equal to ¾ E_{k1}. It also means that the law of conversation of energy is not valid for satellites if they change orbits.

Problem with Centrifugal Force

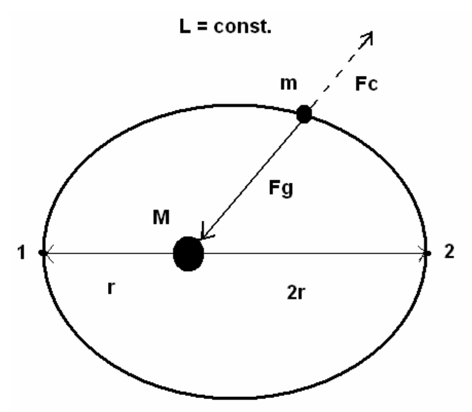
Let's find centrifugal force in orbit 2. It must be in balance with gravitational force again or $F_{c2} = F_{g2}$, so:

$$\frac{mv_2^2}{2r} = \frac{GMm}{(2r)^2} \Rightarrow v_2^2 = \frac{GM}{2r} \quad (23)$$

By changing (10) into (23) above formula becomes:

$$v_2^2 = \frac{v_1^2}{2} \Rightarrow v_2 = \frac{v_1}{\sqrt{2}} \quad (24)$$

Above formula is in conflict with formula (17) originated from the law of conservation of angular momentum. Formula (17) says that velocity in orbit 2 must be double smaller in comparison with velocity in orbit 1 because its radius is double bigger and their product must be constant as per law of conservation of angular momentum. Formula (24) demands that velocity v_2 is not double smaller than v_1, but only for about 30%. This also means that centrifugal force will not be able to hold satellite in orbit 2 as tangential velocity of satellite is too small and gravitational force is double bigger than centrifugal force. This means that orbit 2 will deform into an ellipse.



Picture 5

Note that point 1 and point 2 on ellipse on *picture 5* have the same *radius of curvature r* although their distance from the planet is not the same. Because velocity in point 2 is smaller, centrifugal force is also smaller, and because distance is bigger gravitational force is also smaller. Here centrifugal and gravitational forces will be in balance although both are smaller in point 2 than in point 1.

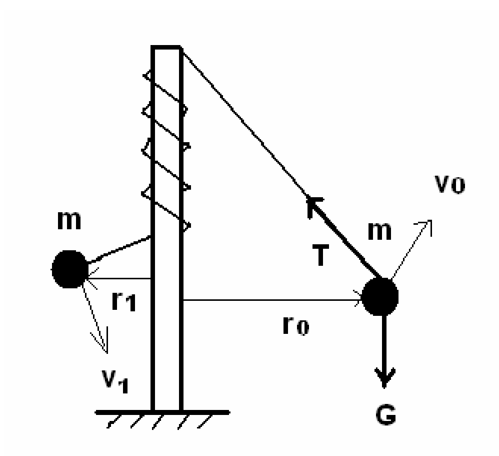
The problem with centrifugal force would exist only if orbit would looks like an ellipsoid (egg shaped orbit) because there radius of curvature would not be the same in point 1 and point 2.

POSSIBLE EXPERIMENTAL PROOF OF OVER UNITY ENERGY

In this chapter will be described experiment with pendulum in horizontal plane. It is winding pendulum and author found it as an example of the law of conservation of angular momentum in university book [4]. It is experiment with stick and thread.

This example from a book of dynamics was given with following description: Small weight with mass *m* is tied to the stick by light and non elastic thread. The weight is on distance r_0 from the stick and it received initial velocity v_0 in horizontal plane. It was necessary to find velocity v_1 when the weight comes to distance r_1 from the stick, see *picture 6*.

Because weight force ***G*** is parallel to the stick it doesn't create moment of force for axis along the stick and doesn't affect velocity in horizontal plane. The same is for tension force ***T*** because it cuts the same axis. This means that the law of conservation of angular momentum (5) is valid for horizontal plane.



Picture 6

Formula for the conservation of angular momentum is down:

$$m\, r_0\, v_0 = m\, r_1\, v_1$$

$$v_1 = (r_0 / r_1)\, v_0 \qquad (25)$$

This was the end of the task in the book, without any further comments. It is very obvious from formula (25) that velocity v_1 will be greater than v_0 because distance r_0 is greater than r_1. If velocity keeps increasing than it is also obvious that kinetic energy will keep increasing by formula bellow:

$$E_{k1} = \frac{1}{2} m v_1^2 = \frac{1}{2} m \left(\frac{r_0}{r_1}\right)^2 v_0^2 \qquad (26)$$

Next question to check was if kinetic energy increased because potential energy was decreased and that way the law of conservation of energy was valid. Author didn't believe in it because it was obvious that coiling of thread around the stick will pull the weight up and also increase potential energy too.

Author decided to check this. He found big wooden spoon and attached the tread by a screw on top of the spoon and on opposite side of the thread a weight was attached. The spoon was fixed on the table and experiment was ready to be performed, see pictures bellow.

Next, the weight was taken to some distance from the spoon and thrown away to have some initial velocity in horizontal plane.

Picture 7

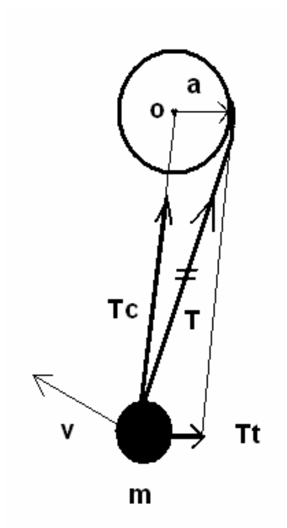
Picture 8

As expected, the weight really went up and velocity was increased first slow than faster and faster. It was very obvious that here both energies were increased and so is total energy.

<u>This would be obvious proof that the law of conservation of energy is not valid where the law of conservation of angular momentum is applicable if there were not starting error in the experiment</u>.

After first release of this document was published, author has received a comment from an expert of mechanics that energy increase was on account of work done by tension force in the thread and that way equations were balanced.

After some correspondence about possible source of energy in the thread, which became shorter and shorter and alone did some work in experiment, it was found that experiment doesn't satisfy the law of conservation of angular momentum. The reason was small but finite thickness of the stick *a,* because it created torque on the stick, but also on the weight. Down is picture with the problem:



Picture 9

Tension force *T* has some angle between center of the rotation *o* and point where the thread touches circumference of the stick because of thickness of the stick *a*. Tension force can be presented as resultant force of two others: Normal force which is also central force T_c and tangential force T_t which has direction opposite of the velocity *v*. Tangential force makes torque against center of rotation and keep decreasing velocity v. The stronger tension force *T* becomes, the stronger tangential force T_t becomes too. It also becomes stronger with shortening of the thread because angle between the thread and center o is becoming greater.

<u>In order to confirm if this experiment has any conservation of angular momentum velocity of the weight must be measured precisely. It can not be decided visually, because constant velocity with shorter thread will have bigger angular velocity which can easy deceive eyes in order to think that tangential velocity has increased</u>.

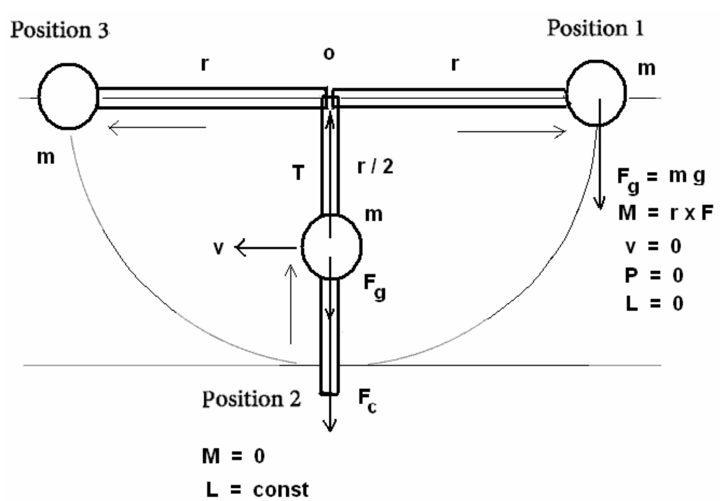
If it becomes proven that there is no any increase of tangential velocity, then this example should be deleted from any book as illustration of the law of conservation of angular momentum. It could only serve as trick question for best students.

PENDULUM AS PARAMETRIC OSCILLATOR

A parametric oscillator is harmonic oscillator whose parameters vary in time. It is said that if parameters vary roughly twice the natural frequency of oscillator, the oscillator absorbs the energy at a rate proportional to energy it already had [5]. In example from *picture 1* a child muscles supplied energy the swing absorbed.

Potential energy of the pendulum bob raised to height *r* is *m g r*. Potential energy would start converting to kinetic energy once the pendulum bob was allowed to fall freely. Conversion is finished when pendulum comes to low position 2. At that position velocity of the pendulum bob is in its maximum. When pendulum starts to raise up it will start to convert some of its kinetic energy to potential energy again. That process of energy conversion would be without the end if friction in pendulum axis and resistance of the air wouldn't exist.

Down on *picture 10* is displayed pendulum with movable pendulum bob.



Picture 10

Kinetic energy at low position 2, before moving pendulum bob up is equal to potential energy in position 1 or position 3:

$$E_{K0} = \frac{1}{2} m v_0^{\,2} = m g r \qquad (27)$$

At position 2 tension force *T* in pendulum handle is also at its maximum and equal to the sum of the weight F_g and centrifugal force F_c. Its formula is:

$$T = mg\,(3\cos(\varphi) - 2\cos(\varphi_o)) \qquad (28)$$

where φ is angle of pendulum handle from vertical line and φ_o is starting angle.

For starting angle of 90 degrees (position 1) tension force in low position 2 (where $\varphi = 0$) is equal to:

$$T = 3\,m\,g \tag{29}$$

Tension force T is in balance with the weight F_g and centrifugal force F_c which keep pressing pendulum bob downwards. Centrifugal force equals to 2mg.

Tension force T can not create torque for pivot point *o* because it cuts it. Only weight F_g can create torque for pivot point *o*. At low position 2 torque created by weight F_g is zero and the law of conservation of angular momentum is valid there.

Angular momentum at position 2 before change of r can be found by formula (5):

$$L = r\,m\,v_0 \tag{30}$$

where v_0 is velocity of the pendulum at position 2, before change of r.

Let's now analyze what would happen if length of the handle r was shorted half of its original size by pushing pendulum bob upwards as on *picture 9*. By using formula (5), angular momentum after changing r to ½ r is:

$$L = \tfrac{1}{2}\,r\,m\,v_1 \tag{31}$$

where v_1 is velocity of the pendulum at position 2, after change of r.

Angular momentum L before and after changing r must stay the same, because of the law of the conservation, and formulas (30) and (31) give:

$$v_1 = 2\,v_0 \tag{32}$$

Kinetic energy after changing r is given as:

$$E_{K1} = \tfrac{1}{2}\,m\,v_1^{\,2} = \tfrac{1}{2}\,m\,(2\,v_0)^{\,2} = 4\,E_{K0} \tag{33}$$

It can be seen that kinetic energy after changing length of the handle r to half of its original size has been increased four times because velocity was doubled in order to conserve angular momentum.

In order to find energy balance, next to be calculated is energy invested into the system in order to push mass m up for the half of the length of r.

Problem with shortening of pendulum handle is increased centrifugal force. By using formula (8) and (32) centrifugal force after pushing bob up is:

$$F_c = \frac{mv_1^2}{r/2} = \frac{m(2v_0)^2}{r/2} = \frac{8mv_0^2}{r} \tag{34}$$

Tension force is equal to the sum of centrifugal force and weight and equals to *9mg* after pushing pendulum bob up. It has been increased three times.

In order to push pendulum bob up by an external force, it should overcome centrifugal force and the weight. Exact mathematics is given in Appendix A and it says that this will cost exactly as much as extra energy given by formula (33).

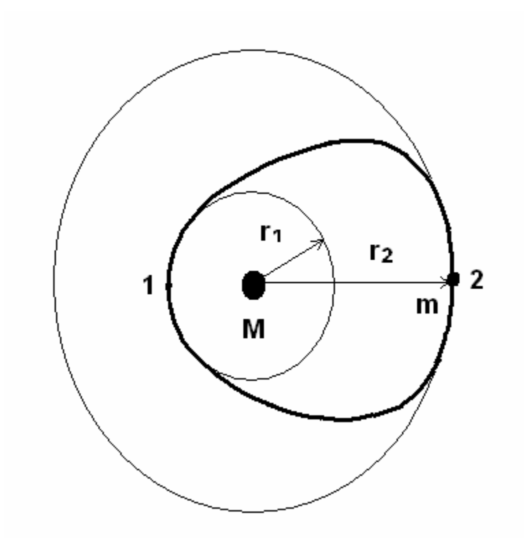
<u>This means that pendulum as parametric oscillator can not give energy surplus by using external force to push pendulum bob up</u>.

If pendulum handle was prolonged than energy surplus would appear, but it would be exactly the same as potential energy increase. That extra energy would be spent to raise the pendulum bob from lower position to end position in order to start new cycle. So, in this case there is no over unity energy too.

IMPORTANCE OF EGG SHAPED ORBITS

We have seen in the problem with centrifugal force that two orbits have problem with energy balance because of disagreement of velocity change. The law of conservation of angular momentum will change velocity linearly; proportional to the change of the distance, and centrifugal force expects proportional change of the square of the velocity.

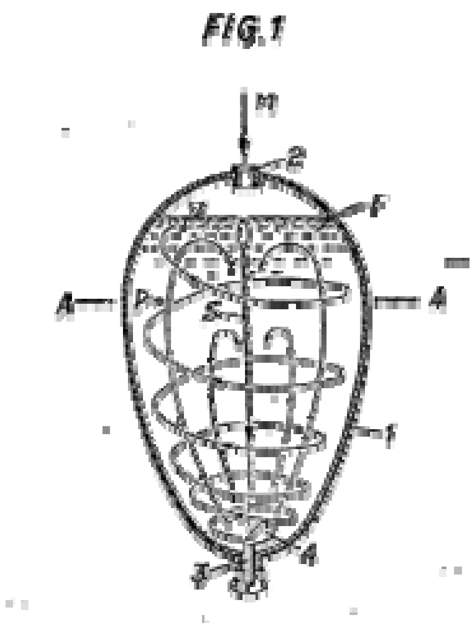
The same problem would also exist if orbit were shaped like an egg because egg shape consists of two orbits, see down on *picture 11*.



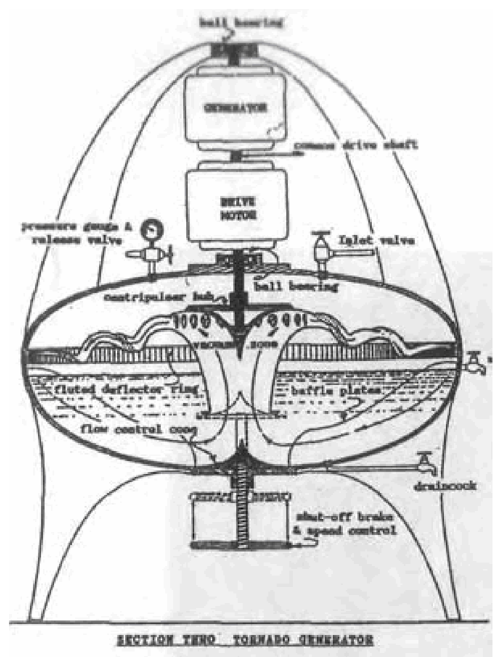
Picture 11

It can bee seen on above picture that radiuses of curvature for point 1 and point 2 are the same as their distances from the center with mass *M*. So, all formulas for two orbits are valid for point 1 and point 2 for egg shaped orbits.

Author doesn't believe that such orbits exists for system with two masses *M* and *m*. However, egg shape exists in nature and is important one. That importance investigated Viktor Shauberger, expert for the usage of the water in many areas and inventor of flying saucer on implosion technology [6]. He used egg shape as it is the only closed shape that will naturally generate vortices, for his repulsator. It was machine built in 1930s and served to convert degenerated water into fresh water with the qualities of mountain spring, see *picture 12*. His implosion motor (Tornado) for generation of electric power directly from water was similar to repulsator and generated ten times more power than motor used to drive implosion engine, see *picture 13*.



Picture 12



Picture 13

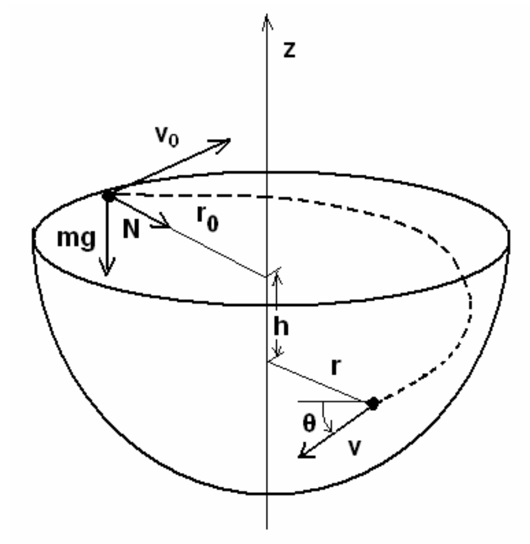
Angular Momentum and Fluids

It is well known fact that any vessel with a hole at the bottom will create vortex in the vessel if it were unplugged and water allowed to go out. This vortex in the middle of the vessel is regarded as a consequence of the law of conservation of angular momentum.

Although vortex does have finite diameter the problem from *picture 9* for horizontal pendulum doesn't exists here. The reason is that the law of conservation of angular momentum is not caused by pull from the middle of the vessel but by push from its surface.

Down is an example, author found in another university book [7], for a particle inside a spherical container and is sure that it can be applied for every drop of water inside any water vessel.

Small ball with mass m is put inside spherical vessel on its edge. Initial velocity v_0 is given to the ball and has horizontal direction. The task was to find angle θ between velocity and horizontal line in a low point, see picture bellow.



Picture 14

We will not calculate the angle here, but only to show that the law of conservation of angular momentum for *z* axis is valid. The reason is that weight force is parallel to the axis and doesn't create torque to the axis. Because spherical vessel is smooth it doesn't have friction and reaction force *N* is always normal to the surface of the vessel. It cuts *z* axis all the time and also doesn't create a torque. Formula for conservation of angular momentum is given down:

$$m\, v_0\, r_0 = m\, v\, r\ \ cosine(\theta) \qquad (35)$$

Conservation of energy for the ball gives:

$$½\, m\, v^2 = ½\, m\, v_0^2 + mg\, h \qquad (36)$$

And height *h* can be found from geometric relation

$$h^2 + r^2 = r_0^2 \qquad (37)$$

For a vessel full of the water analysis would be similar but more complex because every drop of the water must be included in calculation.

For over unity claim of implosion technology following should be taken into account. Viktor used vortex technology for all his engines. According to Bernoulli equation energy of horizontal pipe of the water consists of two members, one for hydrostatic pressure and one for kinetic energy of water movement. Pressure is always bigger for pipes if water doesn't flow. Vortex is additional movement of the water and will consume hydrostatic pressure in the pipe as well any closed water vessel. Note that on *picture 12* there is also vertical movement besides horizontal one. Egg shape could be the best for vertical movement because of the problem with centrifugal force.

Bernoulli equation has also a third member. It is for potential energy if there is a difference in height between two ends of water stream.

However, Viktor Shauberger claimed that existence of vortexes in a river would keep the water cold and healthy. This means that vortex can consume temperature of the water besides hydrostatic pressure. This could explain over unity behavior of vortex technology. If Viktor's claim proved to be true, than a fourth member should be added to Bernoulli equation. It would be a temperature member.

CONCLUSION

In this it has been shown that the law of conservation of energy can be violated in parts of a system where the law of conservation of angular momentum was valid. It has been seen that velocity was increased proportionally when the distance of axis of the rotation was shorten. Proportional velocity increase caused increase of kinetic energy by square rule.

If a satellite was moved from a higher orbit to an orbit more close to the planet, centrifugal force would expect velocity increase by square root rule in order to keep balance by opposing centripetal force, in our case gravitational central force. Because increase was bigger, centrifugal force became bigger and caused body to move away and further increase the distance. This caused corruption of circular orbit and creation of elliptic orbit where total energy is in balance.

If orbit were egg shaped than total energy would oscillate. This difference in total energy would look like an over unity energy which has been produced and returned back by resultant force which is the difference between corrupted centrifugal force and central gravitational force. However, even in this case the law of conservation of energy would be valid for complete system which includes the center with mass *M* (the planet or the Sun), because gravitational force is only external force and centrifugal force is so called fictitious force crated on curved path. It would mean that all over unity energy of the satellite would be derived from gravitational force.

Unfortunately, logic of real parametric oscillator can not be used for pumping out gravitational energy. The only shape with over unity behavior is egg shape and is worth further to investigate.

The path of pendulum bob with movable pivot point as found in two stage mechanical oscillator of Veljko Milkovic [8] looks like deformed half circle or egg shape. This means that centrifugal force doesn't behave exactly as in the case of pendulum with fixed pivot point. It is hard to describe this machine mathematically and precise measuring tools should be used to check the claim of existence of over unity gravitational energy.

Because of over unity claims of Viktor Shauberger author is not sure that Bessler was a fraud and believes that he could make the wheel. Interesting ideas to start with are from John Collin's site [2] and idea of Dr. Peter Lindemann with pendulums locked by escapement logic [9]. However, egg shaped orbits of weights is worth to be investigated.

APPENDIX A

Real Parametric Oscillator

In case of real parametric oscillator the change of parameter can not happen instantaneously and the consequence of the change will be felt in the same time of the change. When the handle of pendulum is shorted it will affect centrifugal force and it will increase energy necessary to invest in order to finish the change of parameter in question.

Down will be analyzed the case of change of the distance from r_0 to any distance *r*.

General formula for centrifugal force is given below:

$$F_c = \frac{mv^2}{r} \tag{A}$$

Formula for velocity in the case of change of distance is given by (25) and will be repeated down again:

$$v = \frac{r_0}{r} v_0 \tag{B}$$

Here velocity v_0 is initial velocity for distance r_0 and v is velocity for distance r.

By changing (B) into (A) formula for centrifugal force is:

$$F_c = m\frac{r_0^2}{r^3} v_0^2 \tag{C}$$

Pendulum from *picture 10* has in its low position 2 kinetic energy equal to initial potential energy. From this equation velocity v_0 in low position 2, before change of the distance *r*, can be found as:

$$\frac{1}{2}mv_0^2 = mgr_0 \Rightarrow v_0^2 = 2gr_0 \tag{D}$$

By changing (D) into (C) formula for centrifugal force become:

$$F_c = 2mg\frac{r_0^3}{r^3} \tag{E}$$

Work *Ac* done against centrifugal force F_c to move pendulum bob from distance r_0 to distance *r* is equal to fictitious work of centrifugal force:

$$A_c = \int_{r_0}^{r} F_c \, dr = 2mgr_0^3 \int_{r_0}^{r} \frac{dr}{r^3}$$

$$A_c = -mgr_0^3 \left(\frac{1}{r^2} - \frac{1}{r_0^2}\right) \qquad \text{(F)}$$

Work *Ag* done by weight force *Fg* for movement from r_0 to *r* is:

$$A_g = -m\,g\,(r_0 - r) \qquad \text{(G)}$$

It can be seen that work done by both forces has minus sign in its formula. It is because the movement happened in opposite direction of the direction of forces. It also means that for this movement external force must be applied. That force will spend energy E_{ex} equal to sum of *Ac* and *Ag* with positive sign.

$$E_{ex} = mgr_0^3 \left(\frac{1}{r^2} - \frac{1}{r_0^2}\right) + mg(r_0 - r)$$

$$E_{ex} = mg\frac{r_0^3}{r^2} - mgr \qquad \text{(H)}$$

Kinetic energy of the pendulum bob after the change of the distance is given by formula (26) and will be given down again:

$$E_k = \frac{1}{2}mv^2 = \frac{1}{2}m\left(\frac{r_0}{r}\right)^2 v_0^2 \qquad \text{(I)}$$

By changing (D) into (I) formula for new kinetic energy equals to:

$$E_k = mg\frac{r_0^3}{r^2} \qquad \text{(J)}$$

Energy Balance of Real Parametric Oscillator

Total energy invested into the pendulum is equal to the sum of initial energy E_0 and energy invested to raise pendulum bob upwards to position *r,* so:

$$E_{tot} = E_0 + E_{ex}$$

$$E_{tot} = mgr_0 + mg\frac{r_0^3}{r^2} - mgr \qquad \text{(K)}$$

Kinetic energy of pendulum after raising the bob is given by formula (J). It can easy bee seen that these two energies are not the same. Formula (J) is identical to the second member in formula (K). This means that total invested energy is greater than kinetic energy left into the system after raising pendulum bob upwards. Energy difference is lost energy and equals to:

$$E_{lost} = E_{tot} – E_k \quad \text{(L)}$$

By changing (J) and (K) into (L) energy difference equals to:

$$E_{lost} = m\, g\, (r_0 – r) \quad \text{(M)}$$

If pendulum bob was raised to the half of its original length $r_0/2$ then lost energy would be $\frac{1}{2}\, m\, g\, r_0$.

Lost energy is proportional to the change of the length of pendulum handle and is equal to the change of potential energy of the system. Kinetic energy left into the system will be able to raise shorted pendulum to end position to start new cycle. So, in this case the law of conservation of energy is valid.

Energy Balance of Extended Pendulum Handle

Next will be analyzed case of extended pendulum handle. This time work A_c and A_g will not be negative as movement is in the same direction as corresponding forces. It means that pendulum bob will be moved down by weight and centrifugal force. Invested energy into the system is only initial energy to raise pendulum in starting position E_0. Output energy equals to work done by centrifugal and weight forces plus kinetic energy left into the system:

$$E_{out} = A_c + A_g + E_k \quad \text{(N)}$$

If for example length of the pendulum handle r was extended to $2r_0$ than according to formula (F) work done by centrifugal force equals to:

$$A_c = \frac{3}{4}\, m\, g\, r_0 \quad \text{(O)}$$

According to formula (G) work done by weight force equals to:

$$A_g = m\, g\, r_0 \quad \text{(P)}$$

According to formula (J) kinetic energy left into the system equals to:

$$E_k = \frac{1}{4}\, m\, g\, r_0 \quad \text{(Q)}$$

By changing (O), (P), (Q) into (N) output energy of the system equals to:

$$E_{out} = 2\ m\ g\ r_0 \tag{R}$$

Although it looks like we have double more output energy into the system all this energy will be spent to raise pendulum with double longer handle $2r_0$ to end position. So, the law of conservation of energy is also valid in this case too.

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