

# Energy Harvesting Using Parametric Excitation

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## Introduction

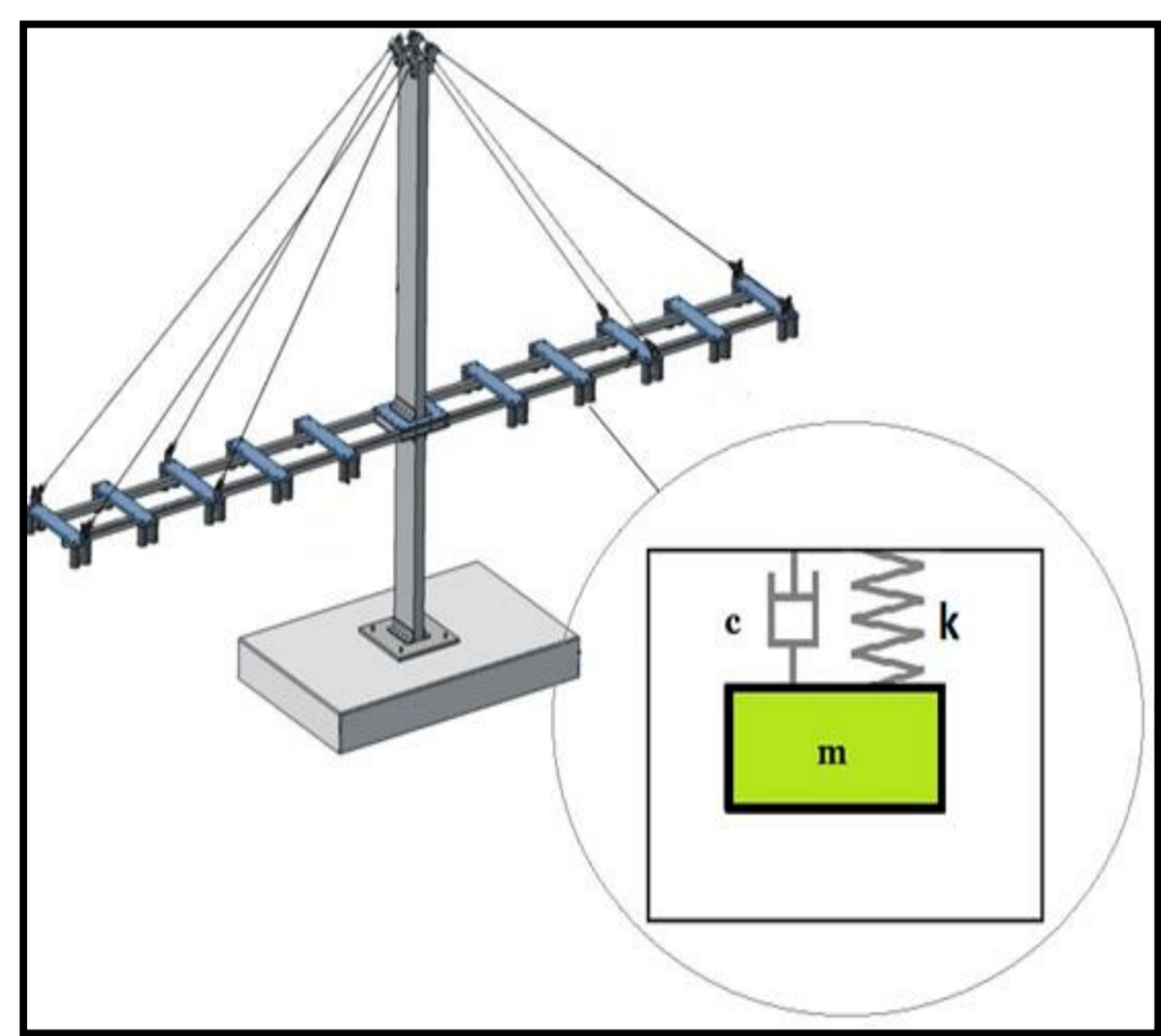
Energy harvesting is the transformation of ambient energy present in the environment into electrical energy. One of the aims of this project is to improve the performance of the linear harvesters by introducing a periodic time-varying co-efficient to a parameter, such as stiffness, to the dynamic equation in order to generate parametric resonance. When the excitation appears as a time-varying coefficient in the equation of motion, the system is parametrically excited. A small parametric excitation can produce a large response when the system is subjected to a frequency equal to double one of its natural frequencies. Parametric resonance has been observed in engineering structures such as ships, cable-stayed bridges and aircraft. A linear harvester on a parametrically excited system (Case 1) and a parametric harvester on a non-parametric system (Case 2) are the cases considered here. The optimum parametric stiffness and parametric frequency are obtained to maximise the peak power.

## What is Parametric Excitation?

A physical system that has a periodic time-variant parameter is parametrically excited. A playground swing-set is a simple example of parametric excitation in a physical system. The swing can be considered as a pendulum whose length changes with time in a periodic manner, since the user of the swing will stand and squat to increase the height of the swing. If the frequency of the periodic oscillation of the swing is equal to twice the frequency of the natural oscillation from the user, the amplitude will increase progressively.

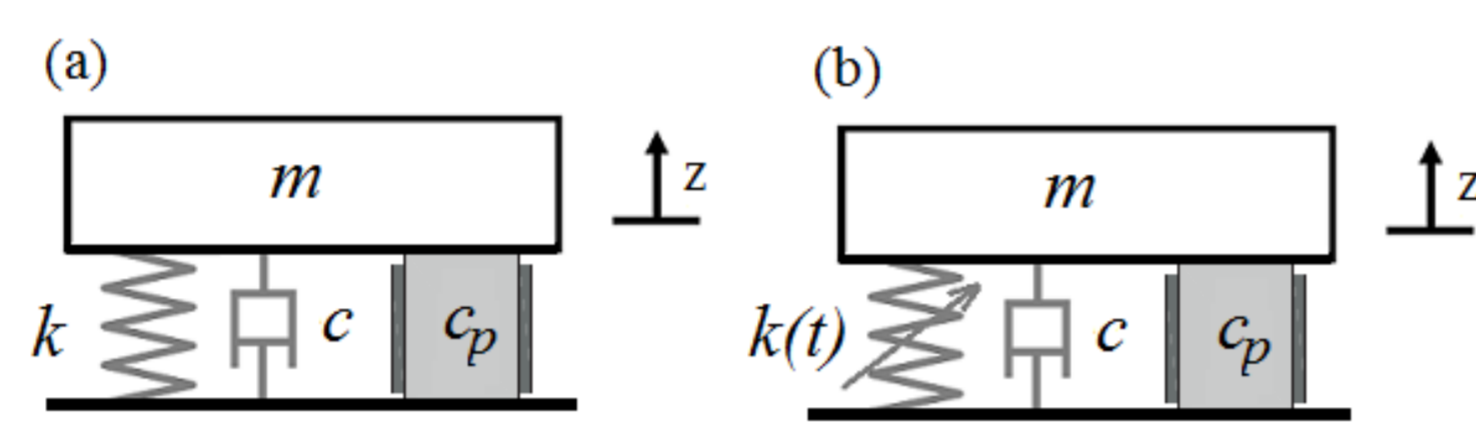
### CASE 1:

#### Parametric Harvester on a Cable-Stayed Bridge



### System Models

Single degree-of-freedom systems with (a) constant stiffness (non-parametric system), and (b) periodic time-varying stiffness (parametric system).

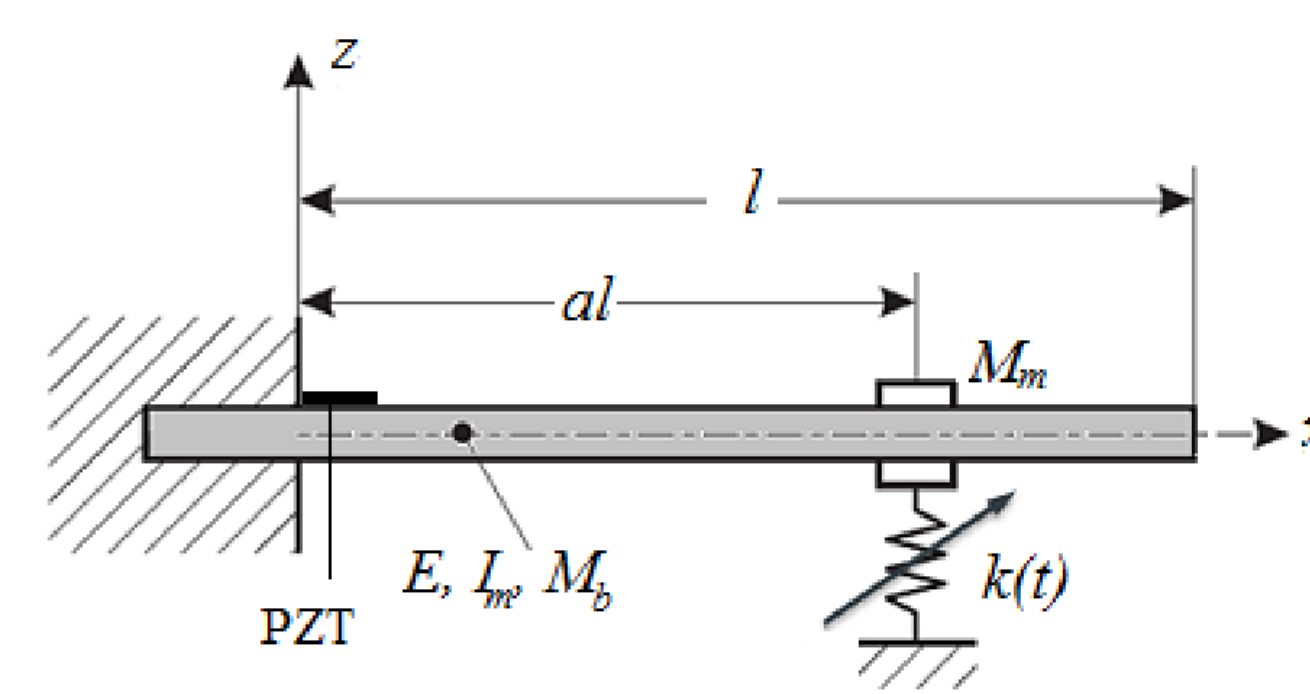


$$m \frac{d^2 z(t)}{dt^2} + c \frac{dz(t)}{dt} + k(t) z(t) = 0$$

$$k(t) = k_c + k_p \cos(\Omega t)$$

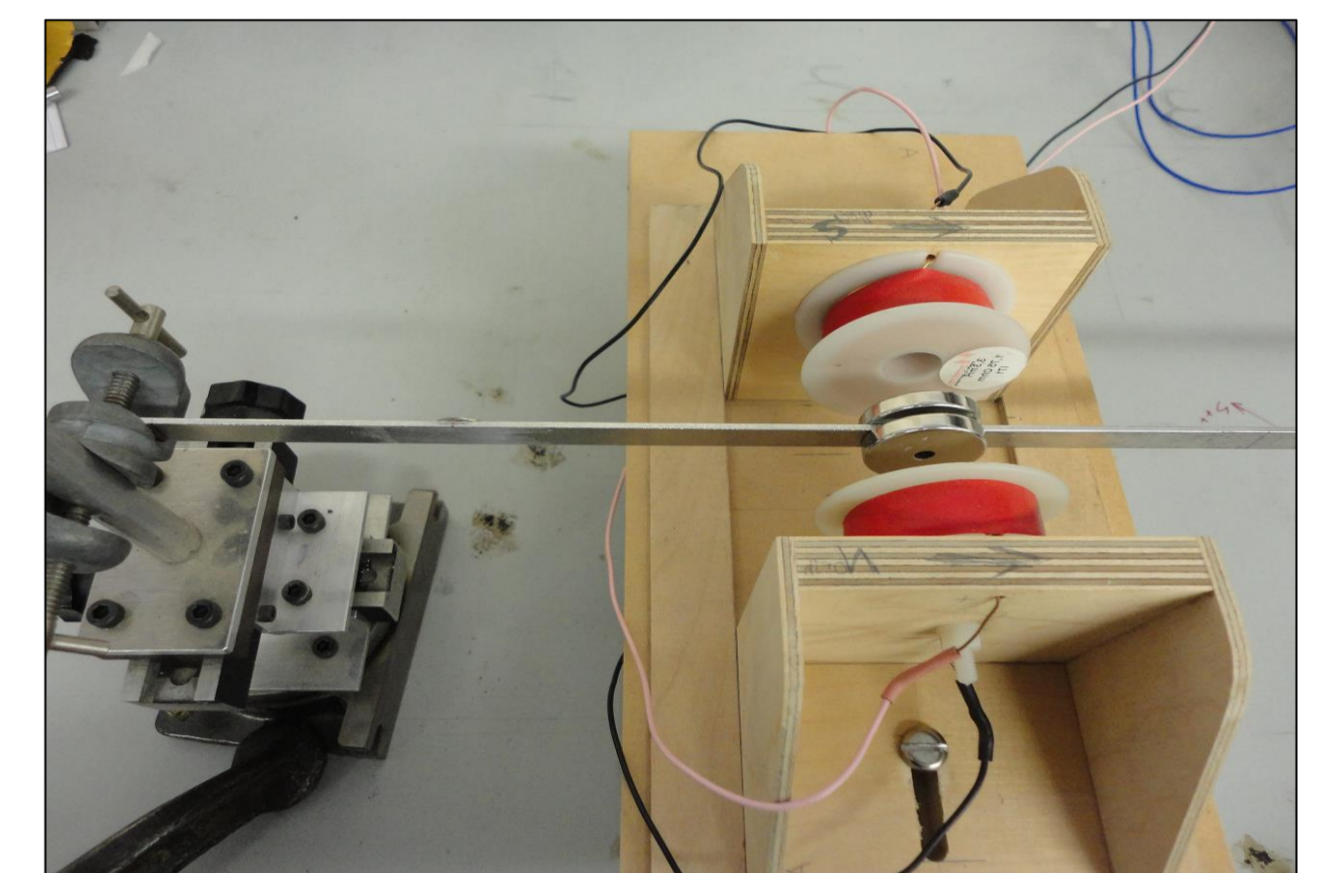
$m$  is the mass,  $c$  is the viscous damping coefficient,  $z(t)$  is the mass displacement and  $k(t)$  is a time-varying stiffness.  $k_c$  is the constant stiffness and  $k_p$  is the parametric stiffness with the parametric frequency  $\Omega$ .  $c_p$  is the piezoelectric element (PZT).

A cantilever beam with an electromagnetic device is modelled as a beam carrying a concentrated mass and a parametric stiffness.

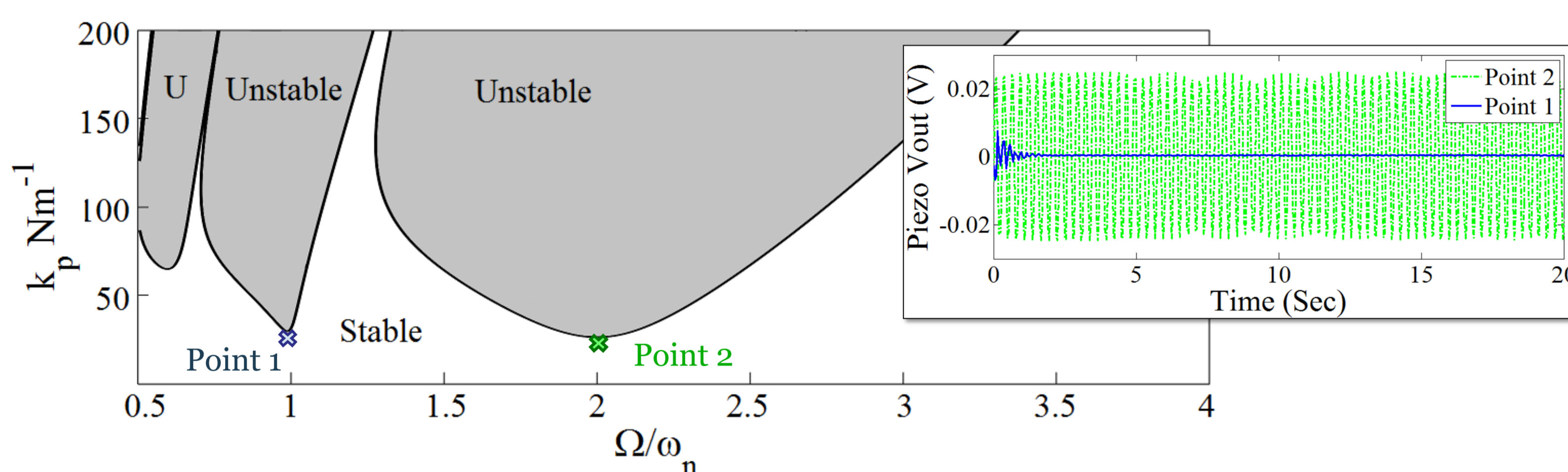


### Experiment

Cantilever beam setup using an electromagnetic device, consisting of two coils and two magnets for changing the stiffness [1].



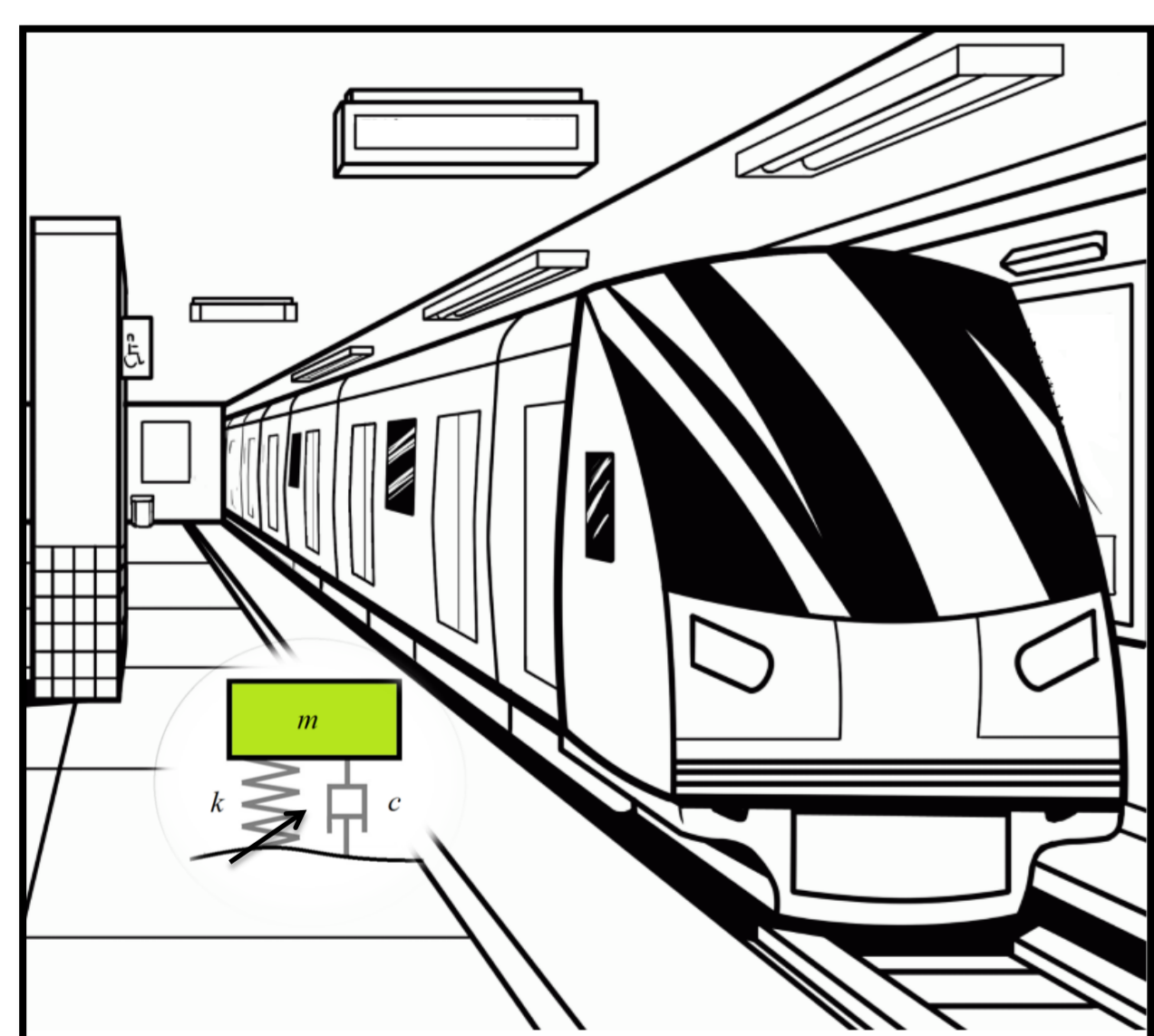
### Stability Chart and Experimental Results



The analytical stability plot shows where the system (b) is stable or unstable for a given value of  $k_p$  and  $\Omega$ . Unstable regions are shaded. Piezoelectric voltage output with the shunt load resistance ( $R = 10 \text{ k}\Omega$ ) for two points in the stability chart are shown. The green graph shows the piezoelectric output when the parametric frequency is twice of the natural frequency and the system is close to instability. The average power calculated at point 2 is a factor 50 greater than point 1. Also, it is a factor 500 greater than the non-parametric harvester with an initial displacement.

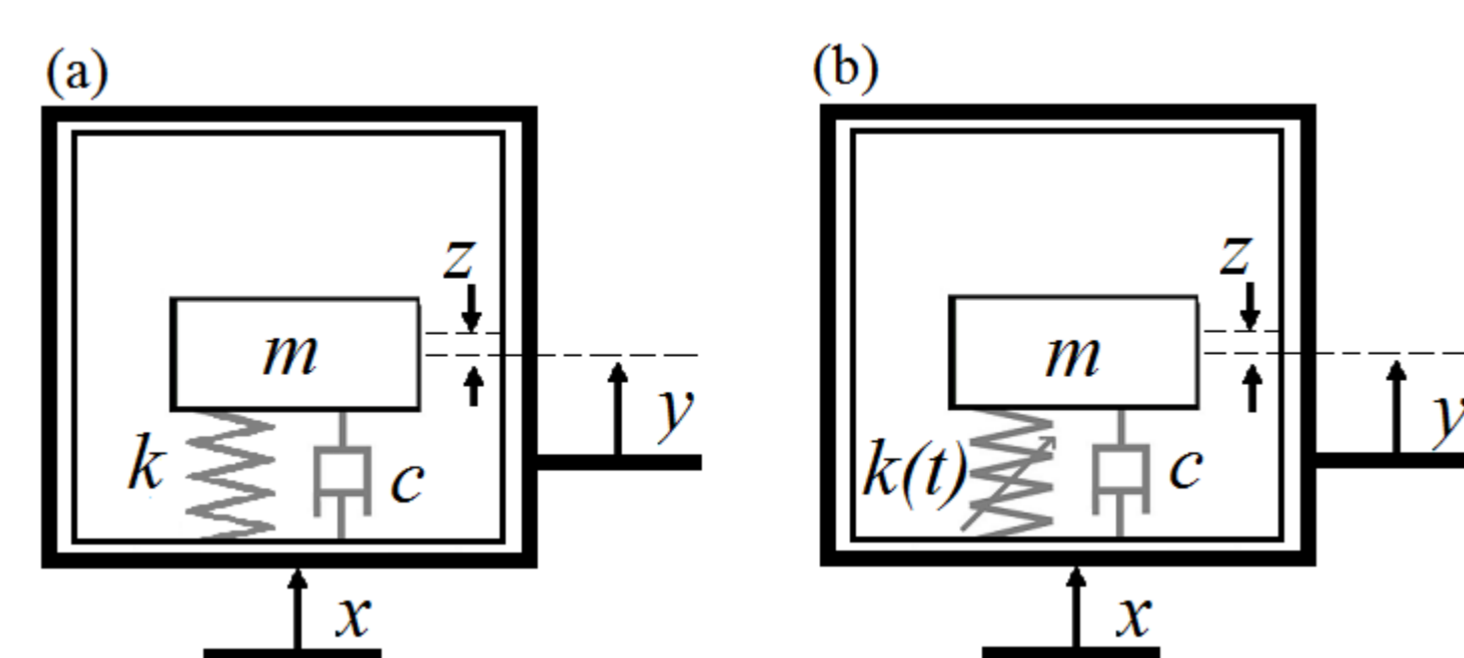
### CASE 2:

#### A Parametric Harvester on a non-Parametric System.



### System Models

Single degree-of-freedom base excited systems with (a) constant stiffness (non-parametric) (b) periodic time-varying stiffness (parametric) are modelled.



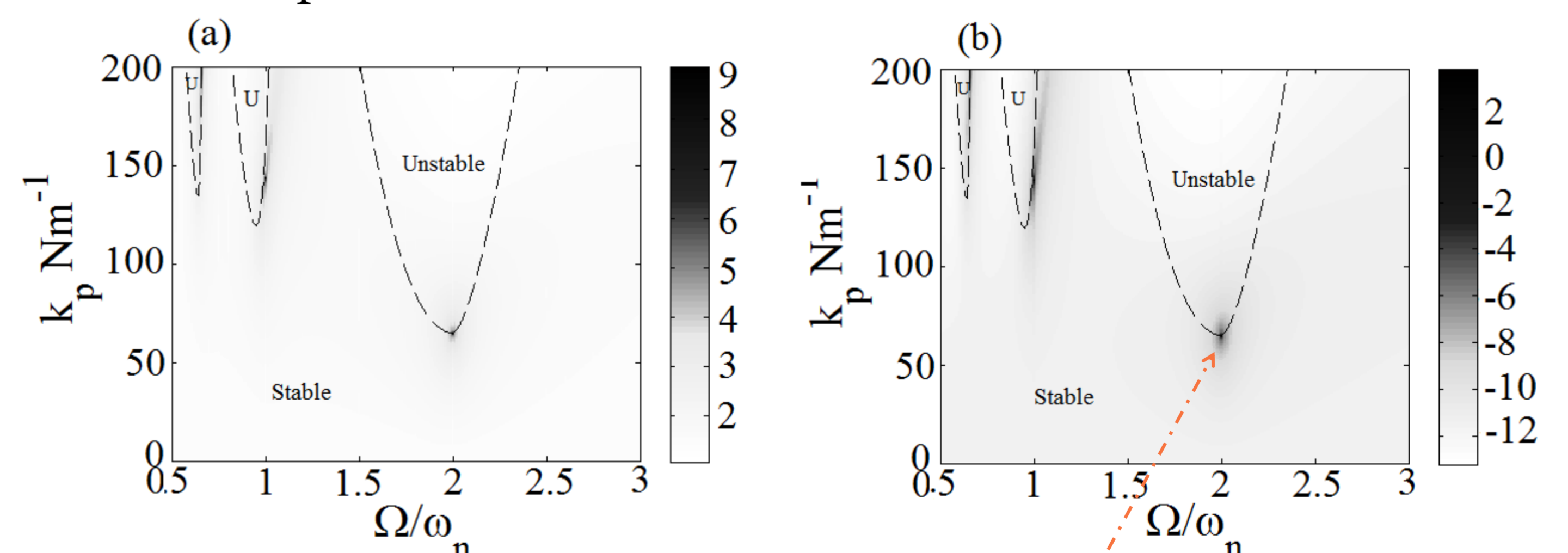
$$z(t) = y(t) - x(t)$$

$$m \frac{d^2 z(t)}{dt^2} + c \frac{dz(t)}{dt} + k(t) z(t) = -m \frac{d^2 x(t)}{dt^2}$$

$$k(t) = k_c + k_p \cos(\Omega t)$$

### Analytical Results

Analytical solution of the parametric harvester subjected to different parametric frequencies and parametric stiffness when the base excitation is equal the natural frequency,  $\omega = \omega_n$ : (a) natural logarithm of transmissibility and (b) natural logarithm of average harvested power.



Maximum power can be harvested when the parametric frequency is twice the natural frequency.

### Conclusion and Future Work

Dynamic characteristics of a parametrically excited system with linear harvester and a parametric harvester are studied analytically and experimentally. For the former, an electromagnetic system is used to generate a periodic stiffness on a cantilever beam. The cantilever beam can behave parametrically by using an electromagnetic system. A piezoelectric sensor was placed in order to harvest energy. The experimental result from the piezoelectric output shows that there is a possibility to harvest 50 times more power when the system is tuned at twice of the natural frequency, and the system which is tuned close to the instability region. As a future work, the latter will be done experimentally.

#### References

[1] F. Dohnal, B. Mace, *Amplification of damping of a cantilever beam by parametric excitation*, (2008).

#### Acknowledgement

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