

## Why is the Integer {1} the First Golden Ratio?

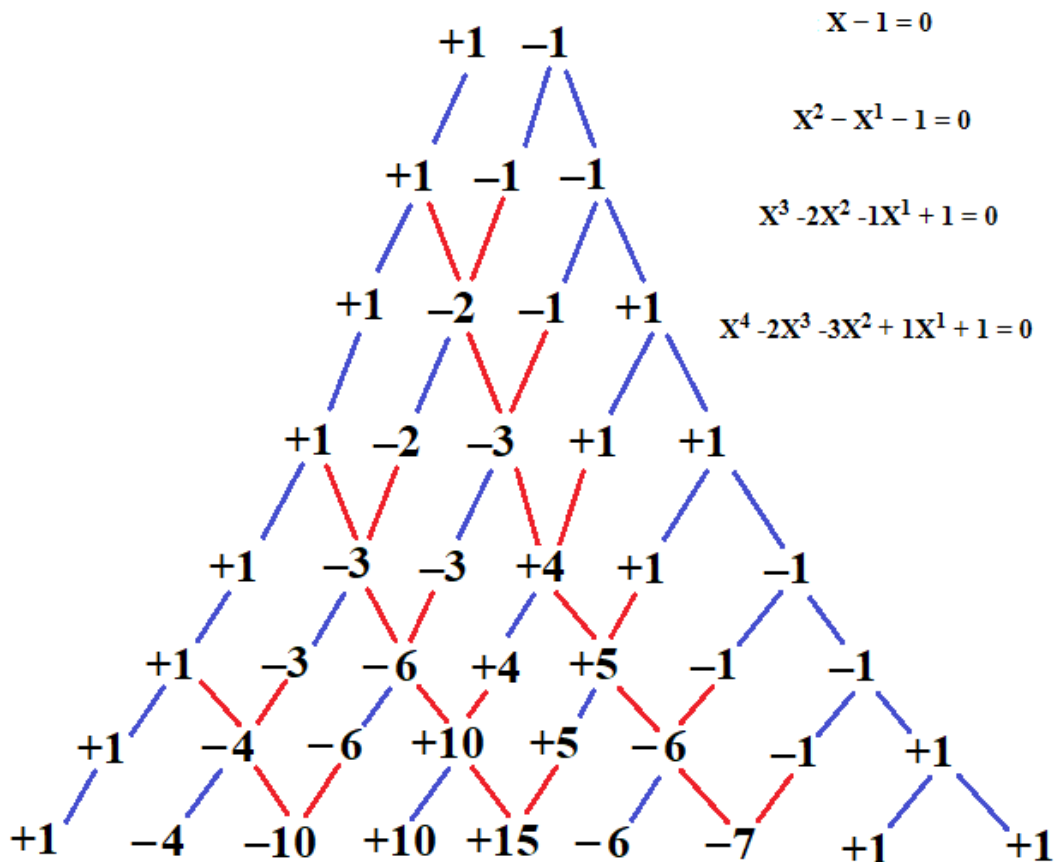
The first Fermat number is the integer: 3. It describes an equilateral polygon in the shape of a triangle. Taking a proportionality of one of its sides divided by its base gives the ratio of the integer: 1. Thus, the integer: 1 is the smallest and the first in a series of five Golden ratios which are constructible and it is the *only* Golden ratio which is also an integer. All of the other Golden ratios are irrational.

The five constructible equilateral polygons – using merely a straightedge and a compass, are: the triangle of three sides, the pentagon of five sides, the heptadecagon of 17 sides, the 257-gon, and the 65,537-gon.

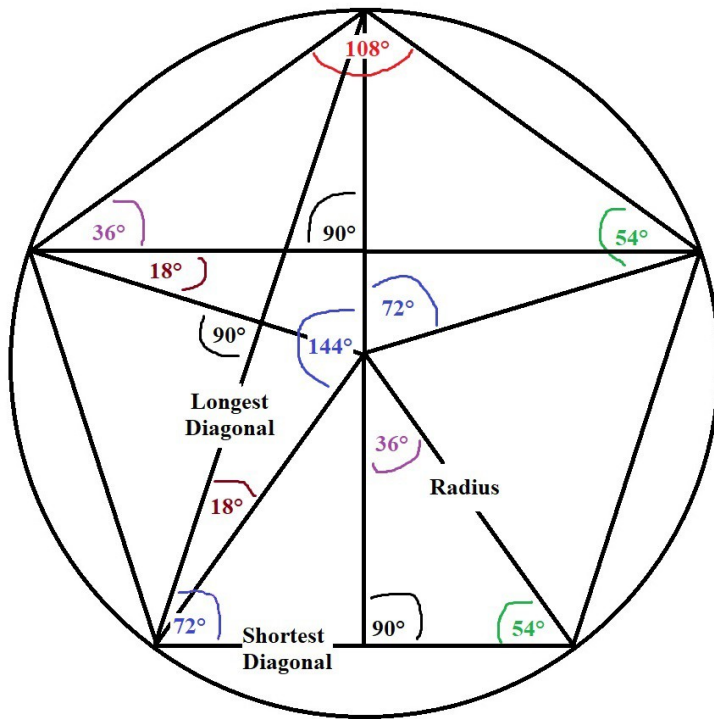
The polynomials in one unknown whose solutions represent these five constructible polygons are:

- $x - 1 = 0 \rightarrow$  yields a triangle with a single solution:  $x = \{+1\}$
- $x^2 + x - 1 = 0 \rightarrow$  yields a pentagon with two solutions:  $x \approx \{+0.618, -1.618\}$
- $x^8 - 4x^7 - 10x^6 + 10x^5 + 15x^4 - 6x^3 - 7x^2 + x + 1 = 0 \rightarrow$  yields a heptadecagon of eight solutions:  $x \approx \{-1.82706, -0.82969, -0.588085, -0.508661, +0.536209, +0.676582, +1.12173, +5.41898\}$ 
  - Source URL: [x<sup>8</sup> - 4x<sup>7</sup> - 10x<sup>6</sup> + 10x<sup>5</sup> + 15x<sup>4</sup> - 6x<sup>3</sup> - 7x<sup>2</sup> + x + 1 = 0 \(Wolfram Alpha\)](https://www.wolframalpha.com/input/?i=x%5E8-4x%5E7-10x%5E6+10x%5E5+15x%5E4-6x%5E3-7x%5E2+x+1%3D0)
  - Shortcut redirect URL: <https://is.gd/ewuder>

The pyramidal shortcut for deriving their coefficients is ...



Here is my cheat sheet for calculating the angles and lengths of diagonals and sides of a pentagon and of a heptadecagon ...



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sides = 5 (pentagon)
(sides - 2) x 180° = 540° total angles
540° total angles ÷ sides = 108° perimeter angle
180° - 108° perimeter angle = 72° radial angle
cos(angle) = adjacent ÷ hypotenuse

radius = 1
perimeter angle ÷ 2 = first cosine angle
108° ÷ 2 = 54°
cos(54) = a ÷ h = a ÷ 1 = a = 0.58778525229247312916870595463907
2 x 0.58778525229247312916870595463907 = 1.1755705045849462583374119092781

1.1755705045849462583374119092781 = shortest diagonal

1.1755705045849462583374119092781^2 = 1.3819660112501051517954131656344
1.3819660112501051517954131656344 - 1 = 0.3819660112501051517954131656344
√0.3819660112501051517954131656344 = 0.6180339887498948482045868343656

radial angle ÷ 4 = second cosine angle
72° ÷ 4 = 18°
cos(18) = a ÷ h = a ÷ 1 = a = 0.9510565162951535721164393337938
2 x 0.9510565162951535721164393337938 = 1.9021130325903071442328786667588

1.9021130325903071442328786667588 = longest diagonal

1.9021130325903071442328786667588^2 = 3.6180339887498948482045868343656
3.6180339887498948482045868343656 - 2 = 1.6180339887498948482045868343656

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longest diagonal ÷ shortest diagonal =
1.9021130325903071442328786667588 ÷ 1.1755705045849462583374119092781 =
= 1.6180339887498948482045868343657

shortest diagonal ÷ longest diagonal =
1.1755705045849462583374119092781 ÷ 1.9021130325903071442328786667588 =
= 0.6180339887498948482045868343657

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sides = 5 (pentagon)

$(\text{sides} - 2) \times 180^\circ = 540^\circ$  total angles

$540^\circ$  total angles  $\div$  sides =  $108^\circ$  perimeter angle

$180^\circ - 108^\circ$  perimeter angle =  $72^\circ$  radial angle

$\cos(\text{angle}) = \text{adjacent} \div \text{hypotenuse}$

radius = 1

perimeter angle  $\div$  2 = first cosine angle

$108^\circ \div 2 = 54^\circ$

$\cos(54) = a \div h = a \div 1 = a = 0.58778525229247312916870595463907$

$2 \times 0.58778525229247312916870595463907 = 1.1755705045849462583374119092781$

$1.1755705045849462583374119092781 = \text{shortest diagonal}$

$1.1755705045849462583374119092781^2 = 1.3819660112501051517954131656344$

$1.3819660112501051517954131656344 - 1 = 0.3819660112501051517954131656344$

$\sqrt{0.3819660112501051517954131656344} = 0.6180339887498948482045868343656$

radial angle  $\div 4 =$  second cosine angle

$$72^\circ \div 4 = 18^\circ$$

$$\cos(18) = a \div h = a \div 1 = a = 0.95105651629515357211643933337938$$

$$2 \times 0.95105651629515357211643933337938 = 1.9021130325903071442328786667588$$

1.9021130325903071442328786667588 = longest diagonal

$$1.9021130325903071442328786667588^2 = 3.6180339887498948482045868343656$$

$$3.6180339887498948482045868343656 - 2 = 1.6180339887498948482045868343656$$

longest diagonal  $\div$  shortest diagonal =

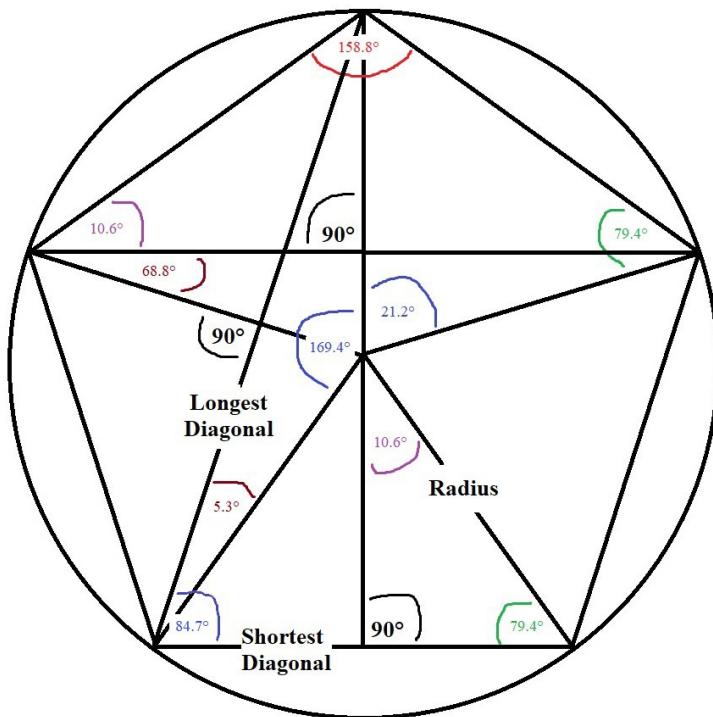
$$1.9021130325903071442328786667588 \div 1.1755705045849462583374119092781 =$$

$$= 1.6180339887498948482045868343657$$

shortest diagonal  $\div$  longest diagonal =

$$1.1755705045849462583374119092781 \div 1.9021130325903071442328786667588 =$$

$$= 0.6180339887498948482045868343657$$



sides = 17-gon  
 (sides - 2)  $\times$  180° = 2700° total angles  
 2700° total angles  $\div$  sides = 158.82352941176470588235294117647° perimeter angle  
 180° - 158.82352941176470588235294117647° perimeter angle =  
 21.176470588235294117647058823529° radial angle  
 cos(angle) = adjacent  $\div$  hypotenuse  
 radius = 1  
 perimeter angle  $\div 2 =$  first cosine angle  
 158.82352941176470588235294117647°  $\div 2 = 79.411764705882352941176470588235^\circ$   
 cos(79.411764705882352941176470588235) = a  $\div$  h = a  $\div$  1 = a =  
 = 0.18374951781657033157440883962073  
 2  $\times$  0.18374951781657033157440883962073 = 0.36749903563314066314881767924146  
 0.36749903563314066314881767924146 = shortest diagonal  
 radial angle  $\div 4 =$  second cosine angle  
 21.176470588235294117647058823529°  $\div 4 = 5.2941176470588235294117647058824^\circ$   
 cos(5.2941176470588235294117647058824) = a  $\div$  h = a  $\div$  1 = a =  
 = 0.99573417629503452187119117890548  
 2  $\times$  0.99573417629503452187119117890548 = 1.991468352590069043742382357811  
 1.991468352590069043742382357811 = longest diagonal  
 -----  
 longest diagonal  $\div$  shortest diagonal =  
 1.991468352590069043742382357811  $\div$  0.36749903563314066314881767924146 =  
 = 5.4189757237297103662675177164563  
 shortest diagonal  $\div$  longest diagonal =  
 0.36749903563314066314881767924146  $\div$  1.991468352590069043742382357811 =  
 = 0.18453671892660399047930221430902

sides = 17-gon

$$(sides - 2) \times 180^\circ = 2700^\circ \text{ total angles}$$

$$2700^\circ \text{ total angles} \div \text{sides} = 158.82352941176470588235294117647^\circ \text{ perimeter angle}$$

$$180^\circ - 158.82352941176470588235294117647^\circ \text{ perimeter angle} =$$

$$= 21.176470588235294117647058823529^\circ \text{ radial angle}$$

$$\cos(\text{angle}) = \text{adjacent} \div \text{hypotenuse}$$

$$\text{radius} = 1$$

$$\text{perimeter angle} \div 2 = \text{first cosine angle}$$

$$158.82352941176470588235294117647^\circ \div 2 = 79.411764705882352941176470588235^\circ$$

$$\cos(79.411764705882352941176470588235) = a \div h = a \div 1 = a =$$

$$= 0.18374951781657033157440883962073$$

$$2 \times 0.18374951781657033157440883962073 = 0.36749903563314066314881767924146$$

$$0.36749903563314066314881767924146 = \text{shortest diagonal}$$

$$\text{radial angle} \div 4 = \text{second cosine angle}$$

$$21.176470588235294117647058823529^\circ \div 4 = 5.2941176470588235294117647058824^\circ$$

$$\cos(5.2941176470588235294117647058824) = a \div h = a \div 1 = a =$$

$$= 0.99573417629503452187119117890548$$

$$2 \times 0.99573417629503452187119117890548 = 1.991468352590069043742382357811$$

$$1.991468352590069043742382357811 = \text{longest diagonal} =$$

$$= 5.0746921161186659373698293977713$$

$$\text{longest diagonal} \div \text{shortest diagonal} =$$

$$1.991468352590069043742382357811 \div 0.36749903563314066314881767924146 =$$

$$= 5.4189757237297103662675177164563$$

$$\text{shortest diagonal} \div \text{longest diagonal} =$$

$$0.36749903563314066314881767924146 \div 1.991468352590069043742382357811 =$$

$$= 0.18453671892660399047930221430902$$

$$8 \times \text{radial angle} = 169.41176470588235294117647058823^\circ$$

$$180 - 169.41176470588235294117647058823^\circ = 10.588235294117647058823529411768^\circ$$

$$10.588235294117647058823529411768^\circ \div 2 = 5.294117647058823529411764705884^\circ$$

$$\text{radial angle} \div 4 = 5.294117647058823529411764705884^\circ$$

$$\text{perimeter angle} \div 2 = 79.411764705882352941176470588235^\circ$$

$$79.411764705882352941176470588235^\circ + 5.294117647058823529411764705884^\circ =$$
$$= 84.705882352941176470588235294119^\circ$$